SCIENCE

Kinematics

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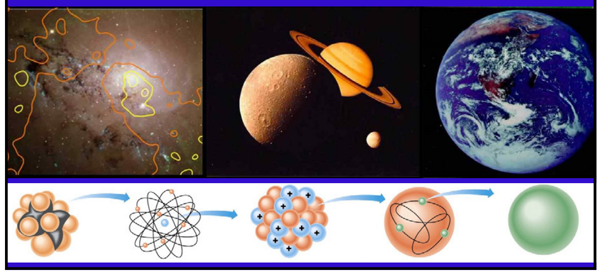
PHYSICS

study of the physical universe
(motion, energy, time, space)
(macro- to micro-cosmos)



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MOTION

- Concept of change
- Motion = change in position over time
- Kinematics (mathematical description)
- Dynamics (explanation)
- Key ideas:
  - cause (force)
  - effect (acceleration)
- Classical mechanics (Newton's Laws)
- Quantum mechanics (subatomic)

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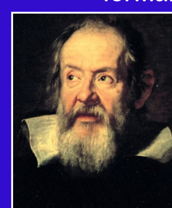
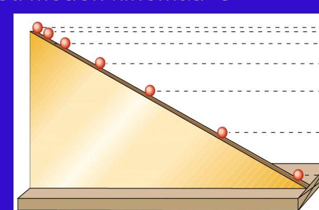
Galileo (1564-1642)

formalized motion kinematics





experimented with position and rates of change

- pre-algebra
- pre-calculus
- used geometry

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Model position and time

- change in position = displacement
  $\Delta s = s_f - s_i$ where f = final, i = initial (m)
- change in time: Δt (s)
- change in position over time = velocity
  $v = \Delta s / \Delta t$ (m/s)
- change in velocity over time = acceleration
  $a = \Delta v / \Delta t$ (m/s^2)

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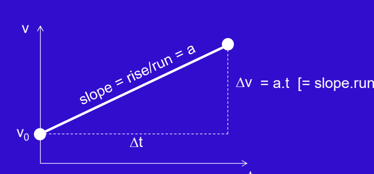
Consider relationships

Consider an object moving with <u>constant acceleration</u> (a)
when time t = 0, object has velocity v_0



After time t, new velocity $v = v_0 + a.t$ [= linear eqn]

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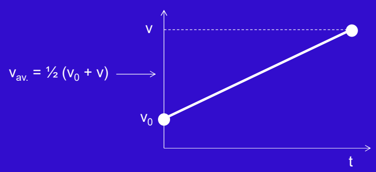
Average velocity

Object moving with constant acceleration

when t = 0, object has velocity v_0 and is at position x_0

when t = t, object has velocity v and is at position x



$v_{av.} = \Delta x/\Delta t \Rightarrow \Delta x = v_{av.}\,\Delta t = v_{av.}\,t$ [as $\Delta t = t - 0$]

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Position

new position $x = x_0 + \Delta x$ [but $\Delta x = v_{av.}t$]

so $x = x_0 + v_{av.}t$ [but $v_{av.} = ½ (v_0 + v)$]

so $x = x_0 + ½ (v_0 + v).t$ [but $v = v_0 + a.t$]

so $x = x_0 + ½ (v_0 + v_0 + a.t).t$

$= x_0 + v_0t + ½ a.t^2$ [which is quadratic eqn]

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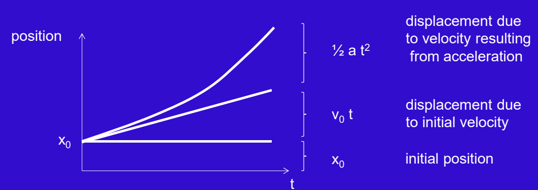
Anatomy of eqn.

$x = x_0 + v_0t + ½ a.t^2$



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Real exemplar

A plane has to make an emergency landing at a small airport with a 900 m runway. The plane touches down at a speed of 80 m/s and then decelerates at 4 m/s².
Will the runway be long enough?

Given data:

touchdown: t = 0; $x_0 = 0$; $v_0 = 80$ m/s; a = -4 m/s²

1. Time until v = 0 ?
   $v = v_0 + a.t = 0$ $t = -v_0/a = -80/-4 = 20$ sec
2. How far in 20 sec?
   $x = x_0 + v_0.t + ½ at^2 = 0 + (80x20) + ½(-4x20x20) = 800$ m

SAFE LANDING!

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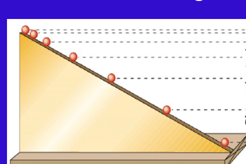
Kinematics (motion)

Galilean kinematics based on geometric logic



Newtonian kinematics includes calculus

- rates of change (differentiation)
- areas under curve (integration)

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Newton (1642-1727)

developed fundamental Laws of Motion

1. INERTIA: an object in motion will remain in motion, and an object at rest will stay at rest, unless acted upon by an external force
   [momentum = mass x velocity]
2. ACCELERATION: acceleration of an object is directly proportional to the force applied,
   and inversely proportional to its mass
   [force = mass x acceleration]
3. REACTION: for every action,
   there is an equal and opposite reaction
   [gravity = 9.8 m.s⁻²]

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Relationships

- rev-head: **"0 to 100 in 10 seconds"**

What does it mean?

Identify and standardize units

- time (t) in seconds (s)
- speed (v) in kilometers per hour (km/hr)
  - convert to meters per sec (m/s):

100 km/hr x 1,000 m/km x 1/(60 x 60) s/hr = 27.8 m/s

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Change in position, velocity, acceleration over time

What variables do we have?

- time interval (Δt) = 10 s
- initial velocity (v_0) = 0 m/s
- final velocity (v_f) = 27.8 m/s
  - so calculate acceleration

$a = \Delta v/\Delta t$

$= (v_f - v_0)/\Delta t$

$= (27.8 - 0) / 10$

$= 2.78\ m/s^2$

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Plot acceleration
$a = 2.78\ m/s^2$

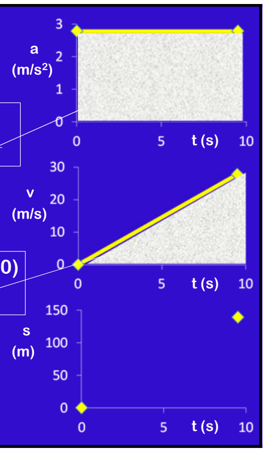
AUC = 2.78 x 10
= 27.8 m/s

Plot velocity
$v_{(0)}$ = 0 m/s
$v_{(10)}$ = 27.8 m/s

AUC = ½(27.8 x 10)
= 139 m

Plot displacement
$s_{(0)}$ = 0 m
$s_{(10)}$ = ??

but what shape?



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Use integration (anti-derivative)

What equations do we have?

a = 2.78 makes sense, constant on graph

$v = \int a$ so $v(t) = 2.78t + C$ ($v_0 = 0$, so $C = 0$)

makes sense, linear graph

$s = \int v$ so $s(t) = 1.39t^2 + 0t + C$ ($s_0 = 0$, so $C = 0$)

makes sense, quadratic graph

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MOTION

derivative (differentiation) ↓

anti-derivative (integration) ↑

	displacement Δs (m)	$s = \int v$
$v = s'$	velocity $\Delta s / \Delta t$ (m/s)	$v = \int a$
$a = v'$	acceleration $\Delta v / \Delta t$ (m/s^2)	

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INTEGRATION

If function $f(x) = x^n$
then integral $F(x) = 1/(n+1)\ x^{n+1} + C$

e.g. $f(x) = 3x^2 + 6x + 2$ $F(x) = x^3 + 3x^2 + 2x + C$

$f(x) = 2e^x$ $F(x) = 2e^x + C$

$f(x) = 0.02e^{0.02x}$ $F(x) = e^{0.02x} + C$

$f(x) = 2e^{0.02x}$ $F(x) = 100e^{0.02x} + C$

Note: $F'(x) = f(x)$

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Let’s play ball!

A ball is thrown vertically into the air at time t = 0 s from a height of 2 m above the ground with an initial velocity of 20 m/s. The acceleration due to gravity at that location is 9.8 m/s^2. Ignore any air resistance.

1. Draw a motion/particle diagram of the ball’s trajectory.
2. What is the velocity of ball at any time t?
3. What is the displacement of ball at any time t?
4. What is the maximum height the ball reaches?
5. When does the ball hit the ground?

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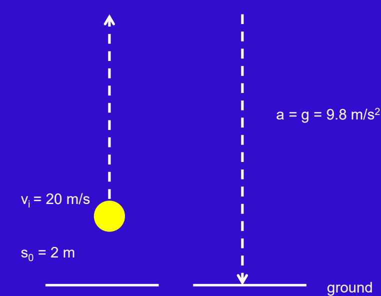
Let’s play ball!

1. Motion/particle diagram of trajectory



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Let’s play ball!

2. What is the velocity of ball at any time t?

v = ∫a

v(t) = ∫a.dt

= ∫-9.8.dt

= -9.8t + C

v(0) = 20 = (-9.8 x 0) + C, so C = 20

giving v(t) = -9.8t + 20

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Let’s play ball!

3. What is the displacement of ball at any time t?

s = ∫v

s(t) = ∫v.dt

= ∫(-9.8t + 20).dt

= (-9.8/2)t^2 + 20t + C

= -4.9t^2 + 20t + C

s(0) = 2 = (-4.9 x 0^2) + (20 x 0) + C, so C = 2

giving s(t) = -4.9t^2 + 20t + 2

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Let’s play ball!

4. What is the maximum height the ball reaches?

maximum height at apex when v = 0

v(t) = -9.8t + 20 = 0

9.8t = 20

t = 2.04 secs

s(t) = -4.9t^2 + 20t + 2

s(2.04) = (-4.9 x 2.04^2) + (20 x 2.04) + 2

= 22.4 m

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Let’s play ball!

5. When does the ball hit the ground?

reach ground when s = 0

s(t) = -4.9t^2 + 20t + 2 = 0

solve for t using quadratic soln

x = (-b $\pm$ $\sqrt{b^2 – 4ac}$) / 2a

t = (-20 $\pm$ $\sqrt{20^2 – (4 \times -4.9 \times 2)}$) / (2 x -4.9)

= 4.179 secs

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Examples

Differentiation		Integration	
Function/relationship	symbols	Function/relationship	symbol
displacement, *s(t)* → velocity, *v(t)*	$v(t) = ds/dt$	velocity, *v(t)* → displacement, *s(t)*	$s = \int v(t)dt$
velocity, *v(t)* → acceleration, *a(t)*	$a(t) = dv/dt$	acceleration, *a(t)* → velocity, *v(t)*	$v = \int a(t)dt$
mass, *m(x)* → density, *ρ(x)*	$\rho(x) = dm/dx$	density, *ρ(x)* → mass, *m(x)*	$m = \int \rho(x)dx$
population, *P(t)* → growth, *r(t)*	$r(t) = dP/dt$	growth, *r(t)* → population, *P(t)*	$P = = \int r(t)dt$

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