SCIENCE

Rocket science (HyShot)

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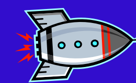
Study of motion

Differentiation and integration essential to

Rocket science



- position (displacement, trajectory)
- velocity (speed, direction (vectors))
- acceleration (due to thrust, due to gravity)

Average values (not so helpful)
Instantaneous (where at any when)
(real-time telemetry)

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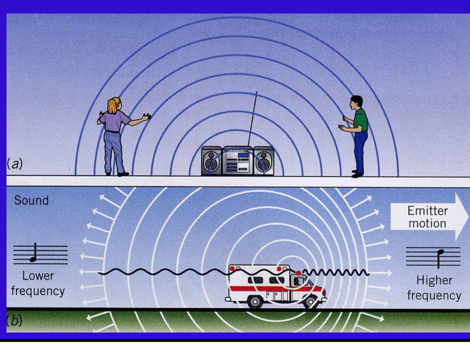
Speed of Sound

Sound waves travel 340 m/s at sea level



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Lightning and Thunder

How far away is the storm? (develop an equation)

Let t = time between flash and bang

know speed of sound (v) = 340 m/s

$s = \int v$
$= \int 340$
$= 340.t + C$ but $s_{(0)} = 0$, so $C = 0$

giving equation $s_{(t)} = 340.t$ (in metres)

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Speed of Sound

340 m/s at sea level (= Mach 1)

subsonic (< Mach 1)
sonic boom (break sound 'barrier')
supersonic (> Mach 1, but < Mach 5)
hypersonic (> Mach 5)

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Speed of sound

convert units from m/s to km/hr for
Mach 1
Mach 3
Mach 5
Mach 20
Mach 30

Mach 1 = 340 m/s x (1 km / 1,000 m) x ((60 x 60 s) / 1hr)
= 1,224 km/hr

Mach 3 = 1,224 x 3 = 3,672 km/hr
Mach 5 = 1,224 x 5 = 6,120 km/hr
Mach 20 = 1,224 x 20 = 24,480 km/hr
Mach 30 = 1,224 x 30 = 36,720 km/hr

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Powered Flight

Propeller engines (< Mach 1)
- petrol engines turn propeller blades

Jet engines (air breathers)
- Turbojets (Mach 3)
  high speed turbine
- Afterburners and Ramjets (Mach 3-5)
  but still subsonic air flow for mixing and combustion
- Scramjet (Mach 5-20)
  supersonic air flow for mixing and combustion

Rockets (Mach 30+)
- fuel and oxidizing agents carried on board

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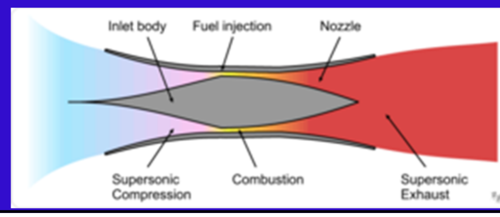
Scramjet

Supersonic Combustion ramjet

essentially, no moving parts
- constricted tube to compress inlet air
- combustion chamber to burn added fuel
- nozzle for exhaust jet
  (leaves at higher speed than inlet air)



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UQ HyShot

Rockets are primarily affected by:
- THRUST (from engines)
- GRAVITY (from celestial bodies)
- DRAG (when moving through atmosphere)
- LIFT (aerodynamics)


HyShot II launch 30 July 2002


HyShot III launch 25 March 2006

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UQ HyShot

video

www.youtube.com/watch?v=nwikJ4wcSEc

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Typical scramjet flight

A scramjet engine is attached to a two-stage rocket and fired to a great height.

After reaching the peak of its trajectory, gravity accelerates the scramjet back towards Earth.

When it reaches a certain high velocity, the scramjet engine fires and the test is conducted.

The engine finally crashes back to Earth.

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Flight log

Time (s)	Action
0	Ignition of stage 1 rocket
0-6	Rocket accelerates at rate equivalent to 22 times that of gravity
6-15	Rocket coasts until ignition of stage 2
15-41	Stage 2 accelerates rocket to 8,300 km/hr
46	Nose cone (with scramjet) separates and continues upwards
46-446	Nose cone continues upwards while reorienting itself for re-entry into atmosphere. Maximum height achieved is 330 km.
446-impact	Nose cone descends using gravity to accelerate it to scramjet ignition velocity, which occurs 35 km above ground. Once scramjet ignites, it burns until it reaches height of 23 km. Scramjet then shuts off and free-falls to Earth

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Scramjet

1. Draw a motion/particle diagram.
2. What is the velocity of the rocket when stage 1 stopped firing at t = 6 sec?
3. Find an expression for the height of the rocket at any time after reaching the maximum height.
4. At what velocity is the scramjet travelling when it fires?
5. What factors have been overlooked in the previous calculations (leading to overestimation of the velocity)?

All perfectly reasonable questions requiring answers

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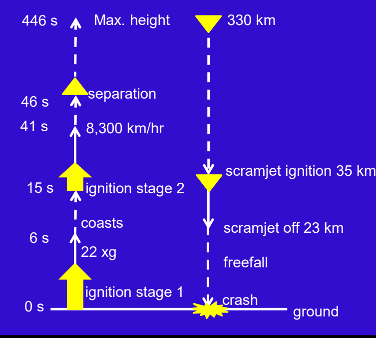
Scramjet

1. Motion/particle diagram of trajectory



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Scramjet

2. What is the velocity of the rocket when stage 1 stopped firing at t = 6 sec?

$v = \int a$

$v(t) = \int a.dt$

$= \int 22g.dt$

$= 22gt + C$

$v(0) = 0 = (22g \times 0) + C$, so $C = 0$

giving $v(t) = 22gt$

$V(6) = 22 \times 9.8 \times 6$

$= 1{,}293.6$ m/s

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Scramjet

3. Find an expression for the height of the rocket at any time after reaching the maximum height.

$v = \int a = \int -g.dt = -gt + C$ $\quad v(0) = 0$, so $C = 0$

$= -gt$

$s = \int v = \int -gt.dt = \int -9.8t.dt$

$= -4.9t^2 + C$

$s(0) = 330{,}000$, so $C = 330{,}000$

giving $s(t) = -4.9t^2 + 330{,}000$ m

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Scramjet

4. At what velocity is the scramjet travelling when it fires?

Fires at 35 km = 35,000 m

so $35{,}000 = -4.9t^2 + 330{,}000$

$t^2 = 60{,}204$

$t = 245$ s

$v(t) = -9.8t$

$V(245) = -9.8 \times 245$

$= -2{,}401$ m/s

= Mach 7.1 (Mach 1 = 340 m/s)

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Scramjet

5. What factors have been overlooked in the previous calculations (leading to overestimation of the velocity)?

- ignored drag (wind resistance)
- change in mass (used fuel)
- change in g with height [g at 330 km = -8.86 m/s^2]

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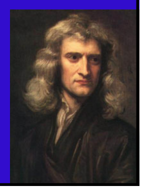
Newton (1642-1727)

developed fundamental Laws of Motion

1. INERTIA: an object in motion will remain in motion, and an object at rest will stay at rest, unless acted upon by an external force
   [momentum = mass x velocity]
2. ACCELERATION: acceleration of an object is directly proportional to the force applied,
   and inversely proportional to its mass
   [force = mass x acceleration]
3. REACTION: for every action,
   there is an equal and opposite reaction
   [e.g. gravity]



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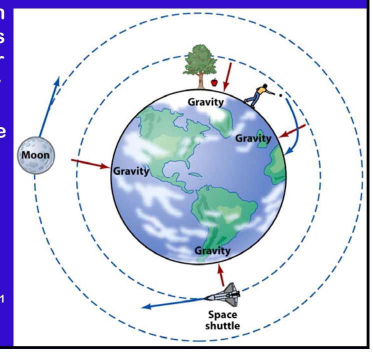
Newton's Law of Universal Gravitation

The force of attraction between any 2 objects is proportional to their masses, and inversely proportional to the square of the distance between them.

$$F = \frac{G \cdot M_1 \cdot M_2}{d^2}$$

Gravitational Constant
$G = 6.672 \times 10^{-11}\ m^3\ s^{-2}\ kg^{-1}$



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Accounting for variable g forces

Let s = scramjet and e = Earth

According to Newton's second law, $F = M_s.a$
According to Newton's universal law, $F = (G.M_s.M_e)/r^2$

thus $M_s.a = (G.M_s.M_e)/r^2$

or $a = (G.M_e)/r^2$

where
a = acceleration due to gravity
r = distance of scramjet from centre of Earth
G = Gravitational constant
M_e = mass of Earth

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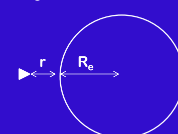
Accounting for variable g forces

6. Write an expression for acceleration due to gravity at any distance r above the surface of the Earth; involving the radius of Earth (R_e) but not G or M_e.

$a = (G.M_e) / (r+R_e)^2$



at surface r = 0
and $a = 9.8 m/s^2$
so $G.M_e = 9.8 (R_e)^2$

hence at any height r,

$a = 9.8 (R_e)^2 / (r+R_e)^2$

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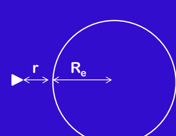
Accounting for variable g forces

7. What is acceleration due to gravity at 330 km above the surface of Earth (highest point reached by scramjet)

$r = 0.33 \times 10^6$ m
$R_e = 6.37 \times 10^6$ m



$a = 9.8 (R_e)^2 / (r+R_e)^2$
$= 9.8 (6.37 \times 10^6)^2 / (0.33 \times 10^6 + 6.37 \times 10^6)^2$
$= 397.65 \times 10^{12} / 44.89 \times 10^{12}$
$= 8.86 m/s^2$

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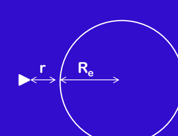
Accounting for variable g forces

8. What is acceleration due to gravity at 35 km above the surface of Earth (height at which scramjet fires)?

$r = 0.035 \times 10^6$ m
$R_e = 6.37 \times 10^6$ m



$a = 9.8 (R_e)^2 / (r+R_e)^2$
$= 9.8 (6.37 \times 10^6)^2 / (0.035 \times 10^6 + 6.37 \times 10^6)^2$
$= 397.65 \times 10^{12} / 41.02 \times 10^{12}$
$= 9.69 m/s^2$

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Accounting for variable g forces

9. Calculate average acceleration due to gravity at heights 330 km and 35 km, and use that to calculate the velocity of the scramjet at ignition.

   at r = 330 km, a = -8.86 m/s^2
   at r = 35 km, a = -9.69 m/s^2
   so average a = [-8.86 + (-9.69)] / 2 = -9.28 m/s^2

thus Q.3 becomes $s(t) = -9.28/2\ t^2 + 330{,}000$
$= -4.64\ t^2 + 330{,}000$
when s = 35,000; t = 252 sec

and Q.4 becomes v(t) = -9.28 t
when t = 252; v = -2,338.6 m/s (~ Mach 6.9)

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Accounting for variable g forces

BUT there is still a source of error!

We have assumed that acceleration due to gravity changed at a constant (average) rate during descent.

Actually, acceleration changes at a different rate during descent (changes according to the square of the distance from the centre of the Earth).

Using integration, the Gravitational law, and the chain rule for integrating as a function of time and displacement, we get:

$$v(r) = -\sqrt{2GM_e\,[(1 / (r+R_e)) - (1 / (H+R_e))]}$$

where v(r) = velocity of scramjet at height r,
and H = initial (highest) height

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Accounting for variable g forces

10. Use this formula to calculate the velocity of the scramjet when it fires at a height 35 km.

$G = 6.672 \times 10^{-11}\ m^3s^{-2}kg^{-1}$ $M_e = 5.97 \times 10^{24}$ kg
$r = 35$ km $= 0.035 \times 10^6$ m $R_e = 6.37 \times 10^6$ m

$$v(r) = -\sqrt{2GM_e\,[(1 / (r+R_e)) - (1 / (H+R_e))]}$$
$$= -\sqrt{(79.6637 \times 10^{13}) \times (0.006875 \times 10^{-6})}$$
$$= -2{,}340.3 \text{ m/s}$$

First estimated at -2,401 m/s (using one g value),
then corrected to -2,338.6 m/s (using average g value),
now corrected to -2,340.3 m/s (using integration)

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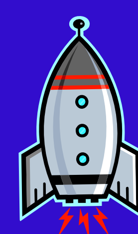
Rocket science

Take home message:

You can obviously see the need for high precision when dealing with rockets, planets, high altitudes, very fast speeds, variable acceleration and big forces.

Mathematically:

$$s' = v = \int a$$



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