Maths in Science

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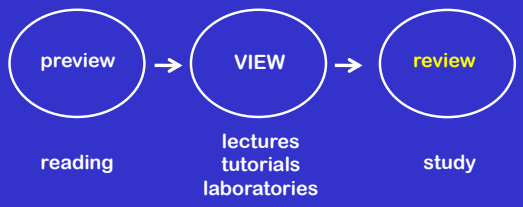
Course design

triple exposure to information



reading

lectures
tutorials
laboratories

study

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Theory and Practice in Science

- thinking
  - philosophy (constantly evolving)
  - reasoning (logic, courage)
- quantitative skills
  - fundamental mathematics (algebra, calculus)
  - modelling (functions, estimation, prediction)
  - computer science (Python)
- contextual relevance
  - natural sciences (biology, physics, chemistry)
  - multiple disciplines, contemporary anecdotes

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Quantitative Models (approximation)

use mathematics to:

- predict change
- understand change
- control change

know I and S, predict O
know I and O, understand S
know S and O, control I

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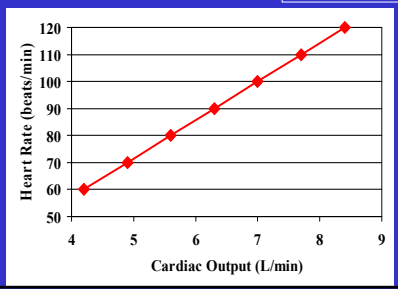
FUNCTIONS

Cardiac Output v. Heart rate
(output increases as heart beats faster)

• linear
$y = mx + c$



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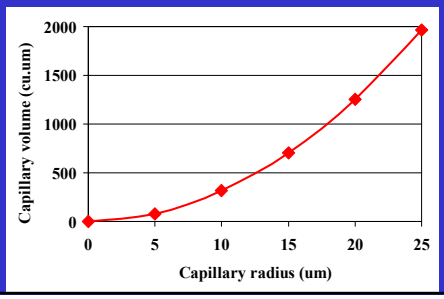
FUNCTIONS

capillary volume v. radius
(volume increases with vaso-dilation)

• quadratic
$y = ax^2 + bx + c$



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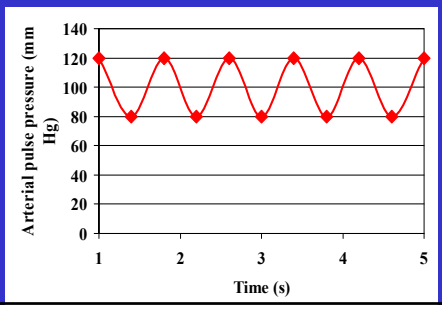
FUNCTIONS

arterial blood pressure v. time
(pulsatile)

• periodic
$y = a.sin(p.x) + e$



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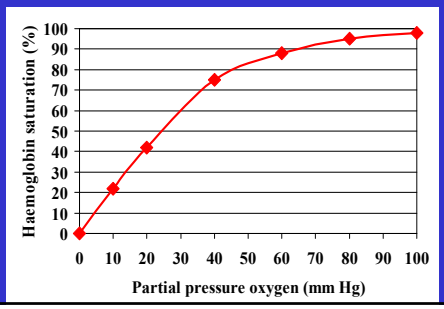
FUNCTIONS

haemoglobin oxygen saturation
(saturation increases with pressure)

• power
$y = ax^p$



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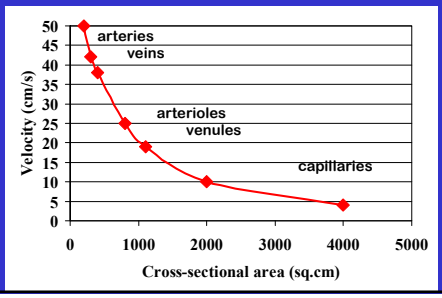
FUNCTIONS

blood velocity v. vessel area
(slower flow in pools than pipes)

• exponential
$y = ce^{-kx}$



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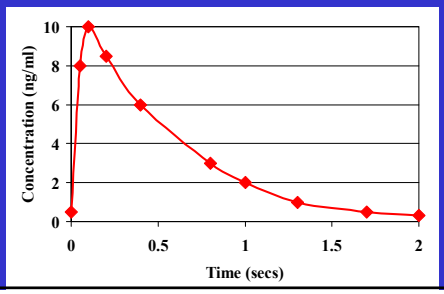
FUNCTIONS

nitric oxide concentration v. time
(potent vasodilator from endothelial cells)

• surge
$y = ax^p e^{-kx}$



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Review - MATHS

FUNCTIONS
(relationships, variables, constants)

• linear	$y = mx + c$	(gradient)
• quadratic	$y = ax^2 + bx + c$	(maxima/minima)
• periodic	$y = a + b.\sin(c.x)$	(equil, ampl, period)
• power	$y = ax^p$	(extrapolation)
• exponential	$y = ce^{kx}$	(growth/decay)
• surge	$y = ax^p e^{-kx}$	(combination)

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Maths in Science

FUNCTIONS	Chemistry/Physics	Biology	Biomedical
LINEAR	Temperature (altitude)		Alcohol (metabolism)
QUADRATIC		Bird distribution (thrush)	Breast cancer (incidence with age)
POWER	Wind chill factor (temp., velocity)	Biodiversity (plant species)	
PERIODIC	Hours of daylight (seasons)		Respiration
EXPONENTIAL	Radio-active isotopes Cooling pH Atmospheric [CO_2]	Algae Bacteria Fish Oysters	Cancer (tumour)
SURGE	Glucose (glycaemic index)		Nicotine Alcohol Antidepressants Contraceptives

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Scientific Anecdotes

Chemistry/Physics	Biology	Biomedical
Temperature (altitude)	Bird distribution (thrush)	Alcohol metabolism
Wind chill factor (temp., velocity)	Biodiversity (plant species richness)	Breast cancer (incidence with age)
Hours of daylight (seasons)	Algae (blooms)	Respiration
Radio-active isotopes	Bacteria (growth)	Cancer (tumour size)
Cooling		Nicotine
pH	Fish (population size)	Alcohol
Atmospheric [CO_2]	Oysters (yield)	Antidepressants
Glucose (glycaemic index)		Contraceptives

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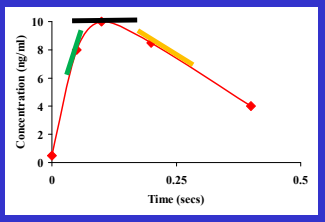
Magic Slope

- dynamic (quantity) + kinetic (time)
  - rate of change (Y') = gradient (slope) = rise / run



$Y = ax^b e^{-kx}$

$Y' > 0$ increase

$Y' = 0$ no change

$Y' < 0$ decrease

- Newton's method (solve for X): $X_{new} = X_{old} - [Y / Y']$
- Euler's method (solve for Y): $Y_{new} = Y_{old} + [Y' . h]$

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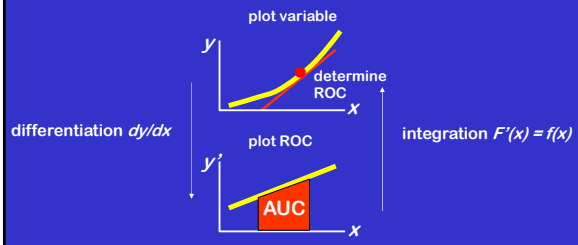
CALCULUS



- Area under curve (total exposure)
- Riemann sums (rectangles), trapezoid rule

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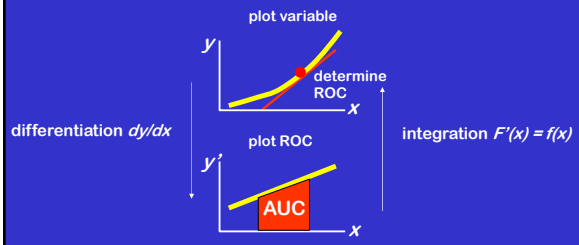
CALCULUS



Fundamental Thereom of Calculus

$$\int_a^b F'(x).dx = F(b) - F(a) \qquad \text{(AUC)}$$

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Differential Equations

- unconstrained (unlimited) [algae, bacteria]
  - exponential growth $Y' = r.Y$

Function $Y = A.e^{r.t}$

- constrained (carrying capacity, C) [fish, oysters]
  - logistic growth $Y' = r.Y\,[1-(Y/C)]$

Function $Y = C.Y_o / [Y_o + (C-Y_o)e^{-r.t}]$

Do not need it! Use Euler's method!

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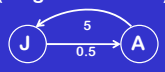
Differential Equations

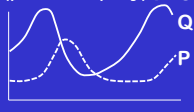
- single population (stage-structured) [fish, turtles]



system of DE's: $J' = 5A - J$

$A' = 0.5J - A$

- two populations (predator-prey) [frogs/crickets, lynx/hare]



- predator population $P' = -cP + dPQ$
- prey population $Q' = aQ - bPQ$

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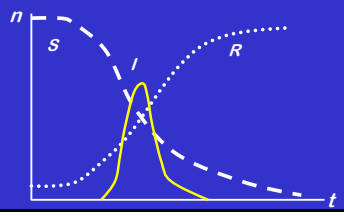
Epidemiology

Model disease in population (compartments) [rubella]

DE's: $S' = -a.S.I/N$ $I' = a.S.I/N - b.I$ $R' = b.I$



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MEDICAL TESTING (and Bayes theorem)

TEST	DISEASE STATUS: present	DISEASE STATUS: absent	
positive	A	B	A+B
negative	C	D	C+D
	A+C	B+D	N

ACCURACY = (A+D) / N
SENSITIVITY = A / (A+C)
SPECIFICITY = D / (B+D)

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Consequences of misdiagnosis

Poor sensitivity
unacceptable number of false negatives

- no treatment → disease progression → death

Poor specificity
unacceptable number of false positives

- unnecessary treatment → side effects → cost

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PREDICTION

Repetitive calculations

- ideal job for computer

- 'off-the-shelf' versus 'do-it-yourself'
- QC/QA 'garbage in, garbage out'

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Review - PYTHON

PROGRAMMING (write your own)

```
from __future__ import division
from pylab import *
```

open, new, save, run module, cut-n-paste

# comments	`# number squared`
I input variable ('prompt')	`a = input ('no.?')`
S expression (function)	`b = a**2`
O output (print)	`print b`

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You get your marks back for 3 assignments. Write a program to calculate your average mark and print your final grade.

INPUT marks (x3)

```
T1 = input ("score for test 1")
T2 = input ("score for test 2")
T3 = input ("score for test 3")
```

SYSTEM calculate av. (formula)

```
T = (T1+T2+T3)/3
```

OUTPUT rank grades (conditionals)

```
if T >= 85:
    print "grade 7"
elif T >= 75 and T < 85:
    print "grade 6"
elif T >= 65 and T < 75:
    print "grade 5"
elif T >= 50 and T < 65:
    print "grade 4"
else:
    print "FAIL"
```

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Choices and Repetition

Conditionals
if, else (elif)

```
if BAC > 0.05:
    print 'Do not drive'
else:
    print 'Go for it'
```

Loops
while

```
i = 1
while i < 5:
    print i
    i = i+1
print 'done'
```

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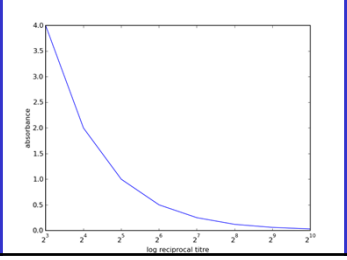
Visualization

graphing

```
x = arange(a,b,s)    # (min,max+1,step)
plot(x,y,'bx')
show()
xlabel('?')  ylabel('?')  title('?')
```



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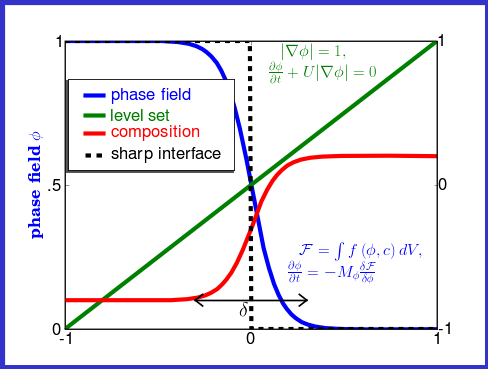
PYTHON – Quo vadit?



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