SCIENCE

NUMBERS

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Science and numbers

Science is descriptive (observational and experimental)
- qualitative (presence/absence, shape, colour…)
- quantitative (number, length, height, weight, mass…)

e.g. density = 1.03 grams/millilitre

numeric value units

value given to 3 significant figures (accuracy/precision)

need to preserve significant figures
if divide by 3, gives 0.343 (not 0.34333333333333333333..

when doing logarithms, preserve sig. figs. in mantissa
log(65) = 1.81 (not 1.81291335..) (only 2 sig.figs.)

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Units

density = 1.03 grams/millilitre

SI base units (m, kg, s, K, mol, A, cd)

Derived units: energy = $kg.m^2.s^{-2}$

Other systems: e.g. Imperial (pounds, feet..)

Unit conversion: factor-label method
convert 20 lb to kg (when 1 lb = 0.45 kg)
20 lb x (0.45 kg / 1 lb) = 9 kg

Special conversions: light year = distance
density = g/mL = g/cc^3

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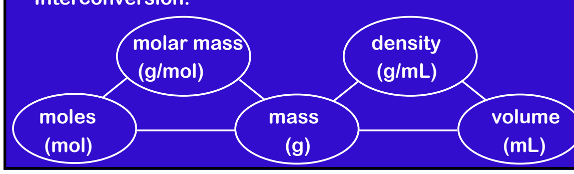
Units

Molecules and moles: 1 mole = Avogadro's number
= 6.022×10^{23} molecules

Atomic mass: 1 amu = 1 g / Avogadro's number
= 1.66×10^{-24} g

Molar mass: 1 M H_2O = 18.02 g/mol

Interconversion:



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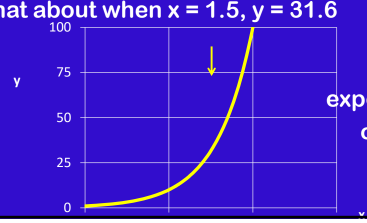
Exponents

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$y = 10^x$ when x = 0, y = 1
when x = 1, y = 10
when x = 2, y = 100

but what about when x = 1.5, y = 31.6



exponential curve

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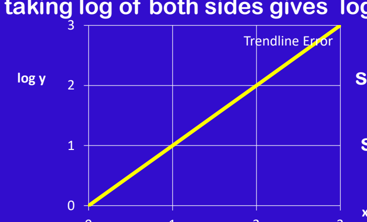
Logarithms (log, ln)

base 10: for function $y = 10^x$
taking log of both sides gives log y = x



semi-log plot gives straight line

base e: for function $y = e^x$
taking ln of both sides gives ln y = x
(e = Euler's number = 2.71828..)

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Orders of magnitude

Density = 1.03 grams/millilitre

= 1.03×10^3 mg/mL

Decimal multipliers:

giga-	$\times 10^9$	(billion)
mega-	$\times 10^6$	(million)
kilo-	$\times 10^3$	(thousand)
milli-	$\times 10^{-3}$	(thousandth)
micro-	$\times 10^{-6}$	(millionth)
nano-	$\times 10^{-9}$	(billionth)

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Orders of magnitude

Let us examine body size:

length / height / diameter (depending on orientation)

SI unit = metre

nano-	micro-	milli-	metres
(billionth)	(millionth)	(thousandth)	(base unit)
10^{-9}	10^{-6}	10^{-3}	10^0

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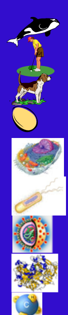
Body size

Many orders of magnitude between:



- whale >10 m
- human
- dog
- egg
- cell
- nucleus
- bacterium
- virus
- protein
- molecule
- atom <1 nm

Let's graph sizes

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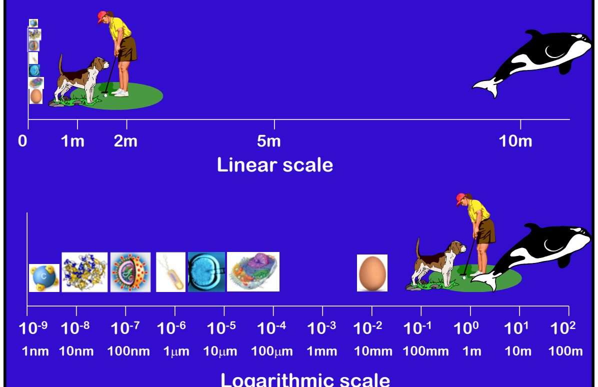
GRAPH BODY SIZE



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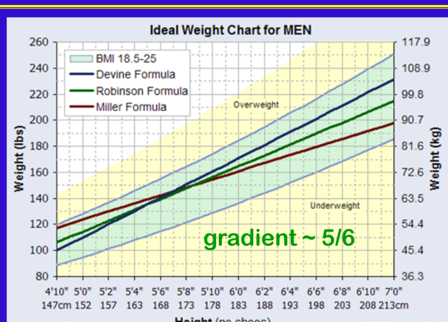
Weight vs height (ideal?)(BMI?)



Isometric or allometric scaling?

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Isometry

Isometric scaling: changes are proportionate (changes in size retain relative proportions)

Governed by square-cube law:

Have cube with L = 1: $SA = 6L^2 = 6$; $V = L^3 = 1$

Double it so L = 2, $SA = 6L^2 = 24$, $V = L^3 = 8$

double (2^1) length → 4x (2^2) surface area → 8x (2^3) volume

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Isometric growth

Poses problems for organisms
- cell / organ / body doubles in length, but now has 8 times volume to support, with only 4 times increase in surface area
- organism has 8 times mass to support, but cross-sectional area only increased 4-fold

Creates mismatch between scaling and physical demands (e.g. elephant is not an up-sized mouse)

Mismatch avoided by:
- being overbuilt when small
- changing proportions during growth (allometry)

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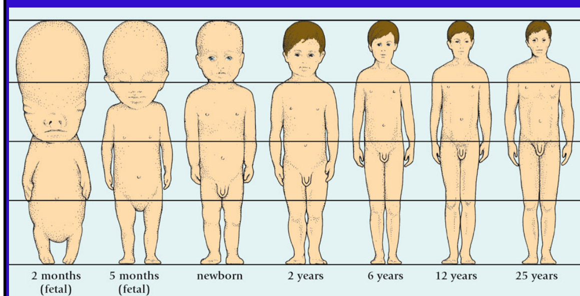
Allometry

Changing proportions during growth



organs/tissues (brain, skeleton, muscles...)

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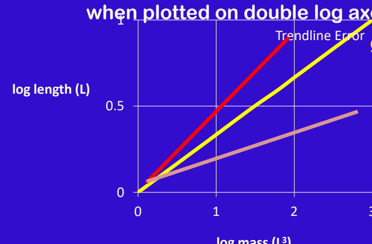
Allometry

Allometric scaling = any deviation from isometry (H_0)

Relationship between length (L) and mass (= L^3) evident when plotted on double log axes



gradient = ?
= rise/run = 1/3

All length-based properties change with mass to 1/3 power

If power > 1/3, then negative allometry (values smaller)

If power < 1/3, then positive allometry (values larger)

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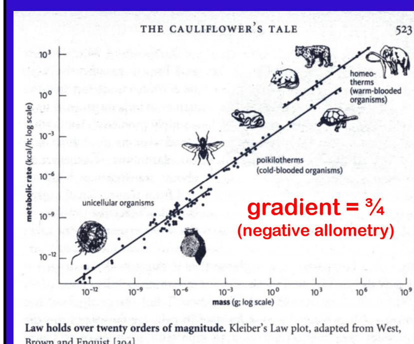
Metabolic rate vs body mass



Law holds over twenty orders of magnitude. Kleiber's Law plot, adapted from West, Brown and Enquist [304].

Kleiber's Law (1932)

BMR = $70M^{0.75}$

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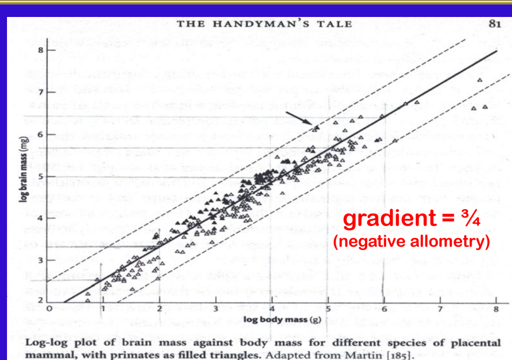
Brain mass vs body mass



Log-log plot of brain mass against body mass for different species of placental mammal, with primates as filled triangles. Adapted from Martin [185].

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Allometry

Numerous examples of allometric scaling

Biology:
- metabolic rate & size
- heat rate & size
- respiration rate & size
- muscle characteristics & size
- bone characteristics & size
- locomotion & size

Engineering
Economics
Ecology
etc......

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Temperature

Numerical measure of hot or cold
(measured by detection of heat radiation or particle velocity or kinetic energy, or by bulk behaviour of thermometric material)

Calibrated in temperature scales in:
- point chosen as zero degrees, and
- magnitudes of incremental units on scale

Celsius scale (°C): empirical scale (centigrade)
0°C = freezing point H_2O, 100°C = boiling point H_20

Fahrenheit scale: empirical
32°F = freezing point H_2O, 212°F = boiling point H_20

Kelvin scale: thermodynamic scale (absolute zero scale)
0 K = -273.15°C or −459.67 °F

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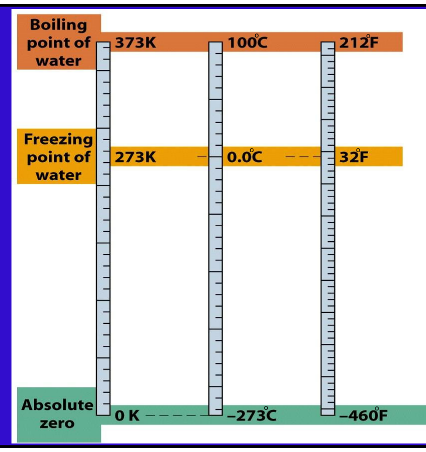

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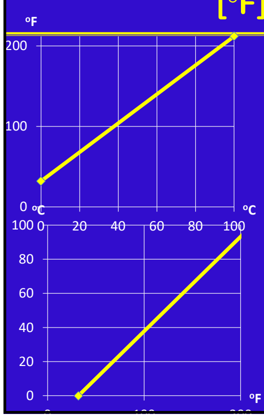
[°F] ↔ [°C]



$y = mx + c$

$m = 180/100 = 1.8$

⇒ [°F] = 1.8 [°C] + 32

$y = mx + c$

$m = 100/180 = 0.56$

⇒ [°C] = 0.56 [°F] – 17.8

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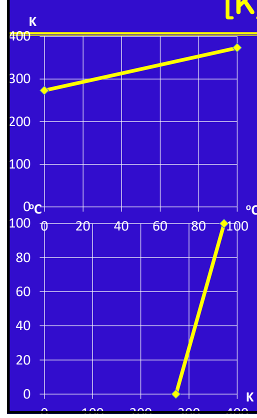
[K] ↔ [°C]



$y = mx + c$

$m = 100/100 = 1$

⇒ [K] = [°C] + 273

$y = mx + c$

$m = 100/100 = 1$

⇒ [°C] = [K] – 273

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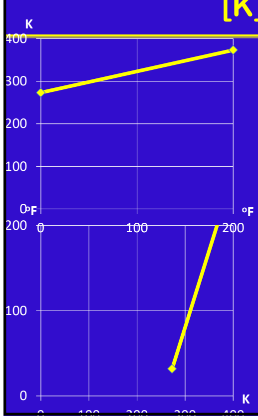
[K] ↔ [°F]



$y = mx + c$

$m = 100/180 = 0.56$

[K] = 0.56 [°F] + 255

$y = mx + c$

$m = 180/100 = 1.8$

⇒ [°F] = 1.8 [K] – 459

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Inter-conversions [?] ↔ [°C]

Fahrenheit	[°F] = [°C] × $^9/_5$ + 32	[°C] = ([°F] − 32) × $^5/_9$
Kelvin	[K] = [°C] + 273.15	[°C] = [K] − 273.15
Rankine	[°R] = ([°C] + 273.15)×$^9/_5$	[°C] = ([°R] − 491.67)× $^5/_9$
Delisle	[°De] = (100 − [°C]) × $^3/_2$	[°C] = 100 − [°De] × $^2/_3$
Newton	[°N] = [°C] × $^{33}/_{100}$	[°C] = [°N] × $^{100}/_{33}$
Réaumur	[°Ré] = [°C] × $^4/_5$	[°C] = [°Ré] × $^5/_4$
Rømer	[°Rø] = [°C] × $^{21}/_{40}$ + 7.5	[°C] = ([°Rø] − 7.5) × $^{40}/_{21}$

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Logarithms

Shown that <u>logarithmic transformations</u> can make power functions appear as linear functions

$y = x^p + c$ becomes: $\log y = p \log x + \log c$

$y = a^x + c$ becomes: $\log y = x \log a + \log c$

<u>Logarithmic scales</u> (display values of physical quantity using intervals corresponding to orders of magnitude)

- Richter scale (earthquakes)
- decibel (sound)
- octave scale (music)
- f-stops (photography)
- entropy (thermodynamics)
- pH (acidity/alkalinity)
- stellar magnitude scale (astronomy)
- Krumbein scale (geology)

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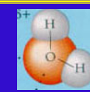
Autoprotolysis of water (ion formation)



$H_2O + H_2O \leftrightarrow H_3O^+ + OH^-$

shorthand $H_2O \leftrightarrow H^+ + OH^-$

Equilibrium constant Kw = $[H^+] . [OH^-]$ = 1×10^{-14}

at equlibrium, $[H^+] = [OH^-] = \sqrt{10^{-14}} = 10^{-7}$ mol/L

Danish chemist Sorenson (1909)

pH = - log $[H^+]$

At equilibrium, pH = - log $[10^{-7}]$ = 7.00

NOTE: units dropped, two sig. figs.

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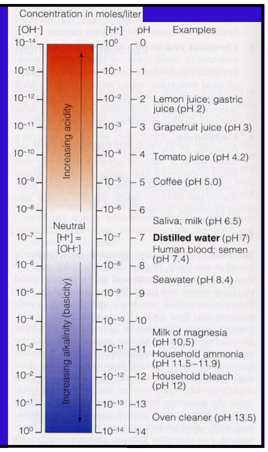
pH scale

Water = fantastic molecule

- polar charge (adhesion/cohesion)
- universal solvent (dissolve electrolytes, sugars, proteins, etc)
- source of protons (hydrogen ions)
- biochemical reactant (hydrolysis)



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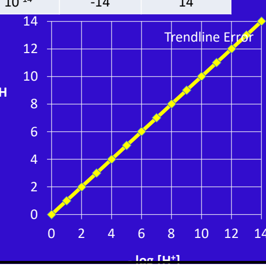
pH = - log [H⁺]

$[H^+]$	$[H^+]$	log [H+]	- log $[H^+]$
1	10^0	0	0
0.1	10^{-1}	-1	1
0.0000001	10^{-7}	-7	7
0.00000000000001	10^{-14}	-14	14



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Science

Quantitation depends on numeric values

- significant figures, multipliers
- orders of magnitude, exponents, logarithms
- units, interconversion

Exemplars

- body size (isometric / allometric growth)
- kinetic energy (temperature)
- hydrogen ion concentration (pH)
- etc
- etc
- etc

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