PARA3002: BioMedical Parasitology

Maths Models

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LIFE on Earth

- chemical basis (carbon-based life on water-planet)
  - proteins, sugars, fats, nucleotides
- genetic code (DNA)
  - replication, transcription, translation
  - four bases (2 bit (binary digit) code)
- cellular organization (membranes, organelles, nuclei)
  - basic units of life
- evolution (natural selection, survival of fittest)
  - mutation, recombination
- symbioses (living together)
  - organelles (SET)
  - organisms (life styles)

  → collective co-existence (ecology)

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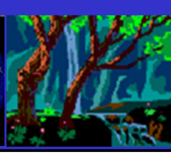
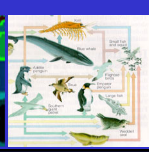
Ecology (hierarchy)

- biosphere (all environments on Earth inhabited by life)
- ecosystems (all living and non-living things within given area) (matter recycles while energy flows through)
- communities (all species within given area) (interactions between species, e.g. food chains, competition, predation, herbivory, disease)
- populations (all individuals of single species) (single species distribution and abundance)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Population ecology

- distribution
  - temporal (any time frame, but esp. seasonal)
  - spatial (any space, but esp. regional)
- abundance
  - size (number)
  - density (number/area)
  - concentration (number/volume)
  - intensity (e.g. number parasites/host)
  - prevalence (e.g. proportion infected)
  - incidence (change in prevalence over time)

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Population growth

Dynamics (numbers) + kinetics (time)

- reproductive population (births)
- senescent population (deaths)

- open population (immigration/emigration)
- closed population (no migration)

- discrete change (step-wise)
- continuous change (constant)

- unconstrained (infinite resources) (exponential)
- constrained (finite resources) (logistic)

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Population characters

- demography (categorize!)
  - biology (age, sex, breed, size ...)
  - other (income, postcode, religion, ...)

↓

- life tables
  - <u>age/stage-structure</u>, numbers
  - births, deaths, migrations

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- survivorship curves
  - life expectancy

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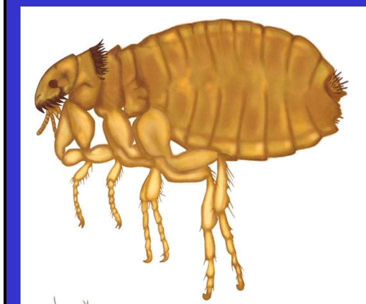
Exemplar: age/stage population structure

Fleas (itchy-scratchy syndrome)



the dangers of not wearing shoes!

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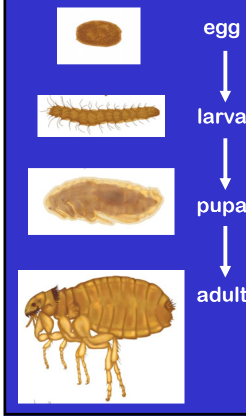
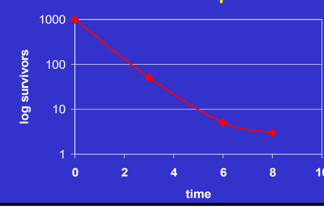
Flea Life History



Life Table

Age (weeks)	Dev. Stage	Number alive	Proportion alive
0-2	eggs	10,000	1.00
3-5	larvae	500	0.05
6-7	pupae	50	0.005
8-20	adults	30	0.003

Survivorship Curve



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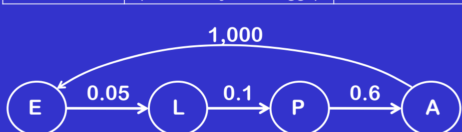
Stage-structured diagrams

- used to depict life-cycles
- arrows show proportion in transition

Time period	Flea developmental stage	Number alive
1	Eggs	10,000
2	Larvae	500
3	Pupae	50
4	Adults (50% female) (females lay 2,000 eggs)	30



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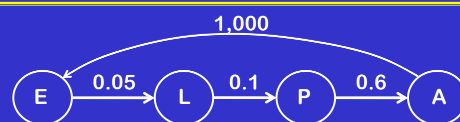
Simultaneous equations



$E_{i+1} = 0\,E_i + 0\,L_i + 0\,P_i + 1{,}000\,A_i = 1{,}000\,A_i$

$L_{i+1} = 0.05\,E_i + 0\,L_i + 0\,P_i + 0\,A_i = 0.05\,E_i$

$P_{i+1} = 0\,E_i + 0.1\,L_i + 0\,P_i + 0\,A_i = 0.1\,L_i$

$A_{i+1} = 0\,E_i + 0\,L_i + 0.6\,P_i + 0\,A_i = 0.6\,P_i$

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Matrix operation

$$\begin{aligned} E_{i+1} &= \big(\, 0\,E_i + 0\,L_i + 0\,P_i + 1{,}000\,A_i \\ L_{i+1} &= 0.05\,E_i + 0\,L_i + 0\,P_i + 0\,A_i \\ P_{i+1} &= 0\,E_i + 0.1\,L_i + 0\,P_i + 0\,A_i \\ A_{i+1} &= 0\,E_i + 0\,L_i + 0.6\,P_i + 0\,A_i \,\big) \end{aligned}$$

transition matrix

$$\begin{pmatrix} 0 & 0 & 0 & 1{,}000 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{pmatrix}$$

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Matrix multiplication

assume population N_0 = 100 E, 50 L, 10 P, 2 A
calculate N_1

$$\begin{pmatrix} 100 \\ 50 \\ 10 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1{,}000 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{pmatrix} \times \begin{pmatrix} 100 \\ 50 \\ 10 \\ 2 \end{pmatrix} = \begin{pmatrix} 2{,}000 \\ 5 \\ 5 \\ 6 \end{pmatrix}$$

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Population growth

- vital statistics of populations (changes over time)
- individuals join (births and immigration)
- individuals leave (deaths and emigration)

$P_2 = P_1$ + (births + immigration) - (deaths + emigration)

- assume closed population (no migration)

$P_2 = P_1$ + (births - deaths)

that is, population is proportionate to:
- current size (P_i)
- growth rate (r) (= birth rate – death rate)

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Rate of change in population (P)

$\Delta P / \Delta t \quad = \quad P' \quad = \quad r \, . \, P$

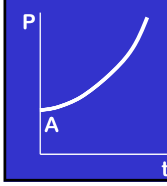
solution is an exponential function

$P \quad = \quad A\, e^{rt}$

differentiate

$P' = r \, . \, A\, e^{rt}$

$\quad = r \, . \, P$



unconstrained growth

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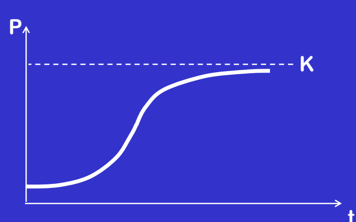
Logistic growth

growth constrained
carrying capacity (K) determined by available resources

$P' \quad = \quad r \, . \, P \, . \, [(K-P)/P]$



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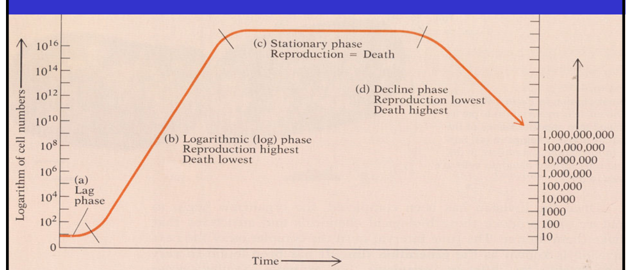
Bacterial population growth



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Epidemiology?

Like ecologists, epidemiologists seek to understand:

– species richness (biodiversity)

– species abundance (populations/communities)

– species distribution (temporal, spatial)

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Epidemiology

Study of occurrence, spread and control of diseases

- Prevalence (number infected)
- Incidence (change in prevalence over time)
- Distribution (density, intensity, concentration,..)

exhibit longitudinal fluctuations (esp. seasonal)

influenced by many factors:
- demographic, socioeconomic, behavioural
- geographic, climatic

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Epidemiological studies

Four main types: | Maths

- Case series (descriptive) — not quantitative
  - index, incidental, miscellaneous
- Case control studies (retrospective) — statistics, Odds Ratio
  - cases + controls interviewed
- Cohort studies (prospective) — statistics, Relative Risk
  - cohort followed forward in time
- Outbreak studies (predictive) — calculus, Differential Equations
  - rate of change in population

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OR and RR

	CASES	CONTROLS
EXPOSED	A	B
UNEXPOSED	C	D

Odds Ratio (OR) = $\frac{AD}{BC}$

Relative Risk (RR) = $\frac{A / (A+B)}{C / (C+D)}$

>> 1 causative?
~ 1 no association
<< 1 protective?

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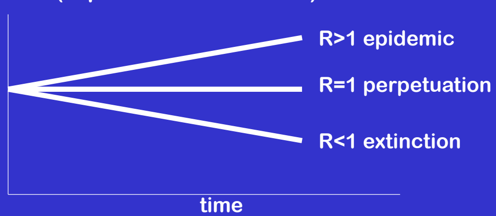
OUTBREAKS

Pathogens must reproduce themselves to survive (reproduction rate = R)



Basic (case) reproduction rate (Ro)
= average number of secondary infections resulting from primary case in susceptible population

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BUT, nothing is IDEAL

Population varies in susceptibility

- proportion just infected (latent)
- proportion patent (infectious)
- proportion immune (resistant)
- proportion lose resistance (susceptible)
- proportion die
- proportion migrate
- etc

⇒ all influence Ro
⇒ need to stipulate model [e.g. SIR model]

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Micro-parasites

viral, bacterial, protozoal pathogens

- reproduce quickly to reach high intensities
- thus causing acute transient infections
- often limited by host immune responses
- recovered individuals develop protective immunity

population dynamics driven largely by transmission rate (contact between susceptible and infected individuals)

- famous SIR model

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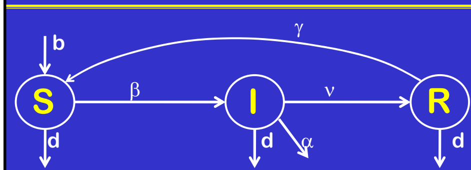
SIR model

(N = Susceptible + Infected + Recovered/immune)



β = infection rate
ν = recovery rate
γ = immunity loss
b = births
d = deaths
α = infection mortality

	in	out
S' =	$bN + \gamma R$	$- dS - \beta SI$
I' =	βSI	$- dI - \alpha I - \nu I$
R' =	νI	$- dR - \gamma R$

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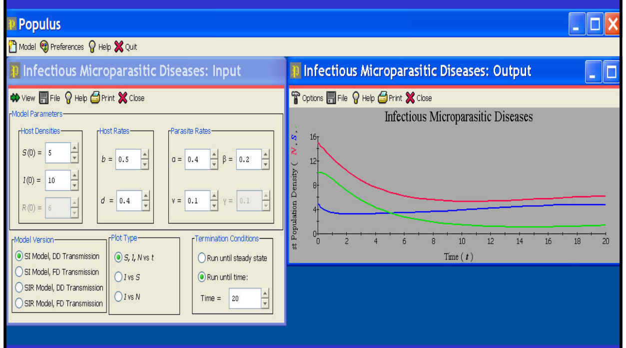

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Macro-parasites

helminths and arthropods

- cause chronic and persistent infections
- disease depends on number present (which in turn depends on exposure to free-living infective stages)

But many infections over-dispersed
(a few hosts have most of the parasites),
so must track intensity of infection

- Anderson & May model: infective stages short-lived
- Dobson & Hudson model: hypobiosis (larval arrest)

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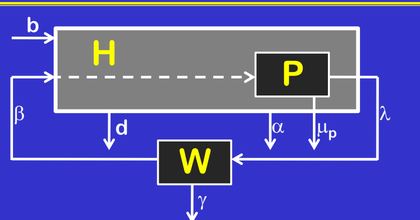
Anderson & May HPW model

Host population, Parasite population (adults), free-living stages (W)



β = transmission rate
λ = parasite birth rate
γ = infective stage mortality
μ_p = adult parasite mortality

b = host births
d = host deaths
α = infection mortality

κ= negative binomial aggregation parameter

	in	out
H' =	bH	- dH - αP
P' =	βWH	- μ_pP - αP - dP - $\alpha P^2/H[(\kappa+1)/\kappa]$
W' =	λP	- γW - βWH

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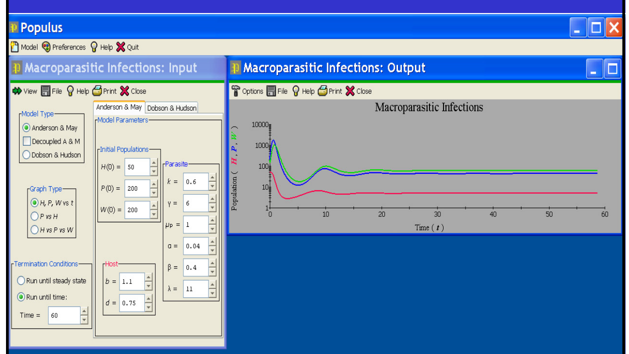

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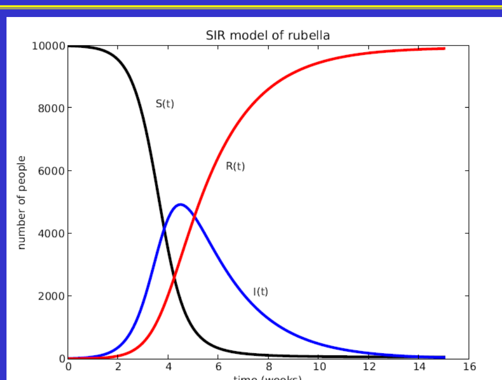
Disease prediction models



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