SCIE1000 Tutorial sheet 10

This tutorial contributes toward your final grade; see the Course Profile (https://www.uq.edu.au/study/course.html?course_code=SCIE1000). The tutorial will be marked out of 6, with 3 marks for completing the "Before class" work, and 3 marks for completing the "In class" assessment and working on the remaining "In class" questions until you finish them or the tutorial ends.

Goals: This week you will work through some general calculation and discussion questions, relating to areas under curves. As usual, you should recognise that the broad concepts and techniques we cover are more important than the specific examples. Do not try to commit lots of facts to memory; instead, know **how** to do things, and **when** certain models and approaches are appropriate. There is no new Python content: from now on we will just revise what we have already covered.

To be completed before class

Complete the following questions before class, write (or type if you wish) your answers on a sheet of paper, put your name and student number on the top of the paper, and hand it to your tutor as you enter the room. **If you do not hand in the answers at the start of the class, as you enter the room, then you will lose the marks for this component.** Note that in some cases there are no "right" or "wrong" answers.

Question (1)

When considering food production, there are times when supply exceeds immediate demand, and other times when demand exceeds immediate supply. Obviously, food shortages can give rise to enormous health, economic and social problems, so it is crucial to try to align food supply with demand.

For many centuries, humans have been preserving food during times of surplus, for later consumption. Traditional preservation techniques included drying, salting, curing, pickling, smoking, cooking and freezing (in cold climates, or with modern technology). Modern preservation makes use of all of those techniques, but one of the most common techniques is to preserve food by canning or bottling.

There are some important requirements for food preservation processes. Clearly, the food must have its nutritional value preserved and be appealing for consumption (in terms of taste, smell, texture and appearance). Most importantly, it must be safe to consume, so not contaminated by bacteria or other pathogenic organisms. Achieving these goals requires balance. For example, many treatments that would kill contaminating bacteria can significantly reduce the nutritional content and palatability of the food.

To prevent microbial contamination, canning processes usually include a *sterilisation phase*. Typically, the sealed cans of food are heated to a "sufficiently high" temperature for a "sufficiently long time" so that there is a very high probability that contaminating bacteria and their spores are killed. By its nature, preserved food is intended to be stored for a significant amount of time, such as years for canned food. If the food is not properly treated then microorganisms can subsequently grow inside the can, spoiling the food and even producing toxins that can lead to food poisoning. For example, *botulism* (https://en.wikipedia.org/wiki/Botulism) is a potentially fatal paralytic illness which can be caused by consuming food containing the *botulinum toxin* (https://en.wikipedia.org/wiki/Botulinum_toxin). This toxin is produced under anaerobic conditions by the bacterium *Clostridium botulinum* (https://en.wikipedia.org/wiki/Clostridium_botulinum)). Botulinum toxin is arguably the most deadly substance known; a concentration of 1 nanogram per kg body mass can be fatal. In theory, 1 kg would be enough to kill the entire human population.

When sterilising food with heat, longer treatment times are required for lower temperatures, and conversely shorter times for higher temperatures. The chosen combination of time and temperature is determined by such factors as the size of the can, the type of food, the pH of the can contents and the types of bacteria that may be present. The following table shows some common contaminating bacteria, and a combination of treating temperatures and times that will (in general) kill 90 % of any of that type of bacteria that are present. In other words, after treatment the size of the population of bacteria will be reduced to 10 % of what it was. In food processing, this is called a *1-Decimal* or *1D* reduction in the population size, and the time taken for this to occur at a given temperature is called the D*-value*. For example, at a temperature of 121 $^\circ$C, the D-value for *Clostridium botulinum* is 12 seconds, so treating food at this temperature for 12 seconds will generally kill 90 % of all individual bacteria.

Organism	**Temperature ($^\circ$C)**	**Time, D-value**
Campylobacter jejuni	55	1 min
Salmonella spp	60	0.98 min
Listeria monocytogenes	71.7	3.3 sec
Escherichia coli	71.7	1 sec

Organism	Temperature ($^\circ$C)	Time, D-value
Staphylococcus aureus	71.7	4.1 sec
Clostridium perfringens	90	145 min
Clostridium botulinum	121	12 sec
Bacillus stearothermophillus	121	5.0 min

Bacteria, temperatures and heating times to kill 90 % of individual organisms.

Clostridium botulinum (https://en.wikipedia.org/wiki/Clostridium_botulinum) is typically used as a "reference organism" for sterilisation processes when canning food. There are several reasons for this. First, as noted above, contamination by *Clostridium botulinum* is potentially fatal, and the botulinum toxin is produced in anaerobic conditions that are typical of canned food. Second, this bacterium is much more resistant to heat than are most of the other common potential contaminating organisms.

The amount of heat treatment received during a sterilisation process is called the F*-value* of the process. The F-value encapsulates both the treatment temperature and the treatment duration. As a general standard reference, the amount of heat treatment received by heating food to a constant temperature of 121 $^\circ$C for a period of 1 minute is defined to equal F-value 1.

In practice, when a population of bacteria is heat treated at a given temperature, the rate at which individuals are killed is proportional to the number of individuals that are present. Thus, the population size follows an exponential decay function over time, with the temperature determining the rate of decay. Noting that an exponential decay function never equals zero (unless it started at zero), this means that it is never possible to **guarantee** that a can of food has no contaminating organisms in it. It is only possible to say that it is **very unlikely** that there are any remaining.

Thus, the standard commercial process is to heat canned food to such an extent that any contaminating *Clostridium botulinum* bacteria and spores are killed with very high probability. The common aim is to use a treatment that would result in a 12D reduction in the size of a population of *Clostridium botulinum*. This is equivalent to a reduction in the population size by 10^{-12}, or a probability of 1 in 1,000,000,000,000 that an individual bacterium will survive the treatment.

From the previous table, treating food for 12 seconds at 121 $^\circ$C gives a 1D reduction for *Clostridium botulinum*, so a 12D reduction can be achieved by treating the food at 121 $^\circ$C for a total of 12×12 seconds which is 144 seconds, or 2.4 minutes. This value is so important that it is commonly written $F_0 = 2.4$. Practical food sterilisation processes must have an F-value at least equal to 2,4 (the value of F_0). In practice, many sterilisation processes will have F-value larger than 2.4, to allow a safety margin.

The previous calculations assume that treatment is to a constant 121 $^\circ$C. This does not mean that 121 $^\circ$C is an optimal temperature for treating, or even a sensible temperature. There are many canned foods for which such a high temperature would damage the food quality. Bacteria will still be killed at temperatures other than 121 $^\circ$C, either more quickly (at higher temperatures) or more slowly (at lower temperatures). Hence the total required heating time may be larger or smaller than 2.4 minutes. The key goal is that the overall F-value of the process must be at least 2.4.

To find F-values at different temperatures, it is necessary to know the rate at which the different temperatures kill bacteria. This rate depends on the temperature and also the particular species of bacterium. The *Lethal rate* L at a certain temperature T is the fraction of the decimal reduction value that is achieved at temperature T

compared to the reference temperature 121 °C, and is given by

$$L = 10^{(T-121)/10}.$$

(There are many good online references that show how this formula is derived. It is based on simple calculations involving exponential functions.) Note that if $T = 121°$C then $L = 1$. In effect, L measures the rate at which bacteria are killed at temperature T compared to what happens at temperature 121 °C, expressed as a fraction (for temperatures below 121 °C) or as a multiple (for higher temperatures).

Then for a given sterilisation process with temperature $T(t)$ at any time, the effective F-value of the process is given by first calculating the lethal rate L for each relevant temperature T, then calculating the area under the Lethal rate curve L considered as a function of time t. As usual, this AUC can be estimated using the trapezoid rule. Then, provided the effective F-value is at least equal to $F_0 = 2.4$, then the sterilisation process can be considered safe.

Consider a can of baked beans subjected to a sterilisation process in which the temperature profile of food in the centre of the can over time matches the data in the following table.

time t (mins)	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Temperature T °C	111	115	118	118	118	115	111

1. Calculate the Lethal rate L for each temperature in that table.

2. Use the trapezoid rule to estimate the F-value of this process (that is, the AUC of the L curve).

3. Is this sterilisation process likely to be safe? Explain your answer.

4. Assume instead that the temperature at time $t = 3$ is 121 °C. What is the new effective F-value, and is the new sterilisation process likely to be safe?

5. By hand, find all of the output produced by the following partial Python program.

```
from pylab import *

def calc(T1, T2):
    v = (T1 + T2)/2
    return v


t = arange(0,7)
Temp = array([111, 115, 118, 118, 115, 111])
i = 2
while i<5:
    m = calc( Temp[i], Temp[i-1] )
    print("Val: ",i, m)
    i = i+1
print("End: ",i, m)
```

6. Show how to modify the program in Part 5 so that it instead calculates and prints the Lethal rate L for each temperature in the array called `Temp`. (Hint: part of your answer will require you to change the function `calc` so that it calculates the L value for a single, given temperature.)End

To be completed in class

Complete the following questions in class. They involve a mix of individual work, and discussions with others. Make sure that you read the questions before class and think about how you might approach answering them. Don't rely on someone else doing all of the work. You need to work by yourself on the final exam, so it is important that you work hard now.

Feedback: Be proactive!

Australian government research shows that students often feel they don't receive adequate feedback on their work. In a class of 800 students, it is not possible for the course coordinator to give direct feedback to each student. Instead, tutorial classes are designed to be the place in which you can get feedback on your work from classmates and the tutors. You can ask for help, show them your answers, and discuss your understanding of any of the course material. As an adult learner, the onus is on **you** to seek feedback; tutors and classmates are happy to give it, if you want it.

Question (2)

Briefly discuss with a partner your answers to Question (1), and, if your answers differ, jointly agree on the correct answers.

Question (3)

(This question was on the final examination in 2012, and worth 8 marks. Expected working time for this question was about 8 minutes.)

Note: When answering this question, do not refer to any of the "Before class" content. You do not need to use Lethal rate or any other concept defined in that work. Simply answer the question as presented.)

In order to be safe for human consumption, many foods need to be heated sufficiently to kill bacteria. Typically, the effectiveness of the heating process is related to both the *temperature* to which the food is raised, and the *time* for which the food maintains that temperature, with higher temperatures required for shorter periods of time to have the same impact.

Trichinosis (https://en.wikipedia.org/wiki/Trichinosis) is a parasitic disease caused by eating raw or undercooked meat, particularly pork, infected with the larvae of a species of roundworm, *Trichinella spiralis* (https://en.wikipedia.org/wiki/Trichinella_spiralis). To help ensure that pork is safe for consumption, the United States Department of Agriculture has published the following table of times for which the internal temperature should be maintained at the given levels.

Temp. ($^\circ$F)	**Time (mins)**		**Temp. ($^\circ$F)**	**Time (mins)**
120	1260		132	15
122	570		134	6
124	270		136	3
126	120		138	2
128	60		140	1
130	30		142	1

Develop a mathematical model of the time t in minutes for which the internal temperature of pork should be maintained at a temperature of $f\,^\circ$F. Show all working and explain your answer briefly. (Hint: your answer should be of the form $t(f) = \ldots.$)

Question (4)

(This question was on the final examination in 2012, and worth 15 marks. Expected working time for this question was about 15 minutes.)

Note: When answering this question, do not refer to any of the "Before class" content. You do not need to use Lethal rate or any other concept defined in that work. Simply answer the question as presented.)

To reduce the risk of *Salmonella* (https://en.wikipedia.org/wiki/Salmonellosis) infections, a company heats egg custard tarts as follows.

> At time $t = 0$ min, tarts have a temperature of 60 $^\circ$C. Their temperature **increases** at a constant rate of 1 $^\circ$C min^{-1} until reaching 70 $^\circ$C. They are held at this temperature for 30 minutes, then the temperature **decreases** at 0.2 $^\circ$C min^{-1} for 10 minutes, and finally **decreases** at 0.8 $^\circ$C min^{-1} for 10 minutes.

For all of this question, assume that *Salmonella* is only killed by temperatures higher than 60 $^\circ$C. The following Python program plots the temperature of the tarts **above the baseline of 60 $^\circ$C**; run the program now, then answer these questions:

1. What does the area under the curve (AUC) of this graph represent and what are its units?

2. Find the AUC of the graph. (Show all working.)

3. In order for the egg custard tarts to be safe to consume, the AUC must be at least 500, in the appropriate units. The only stage of the process that can be changed is the rate of initial heating. (That is, the process must last for a total of 60 minutes, the cooling rates must stay unchanged, and the maximum temperature must be maintained at 70 $^\circ$C.) Find the (constant) rate of initial heating from 60 $^\circ$C to 70 $^\circ$C that will give an overall AUC of exactly 500.

In [ ]:
```
# Program to plot the temperature profile of an egg tart

from pylab import *

t=array([0,10,40,50,60])
temp=array([0,10,10,8,0])
xlabel("Time (mins)",fontsize="large")
ylabel("Temperature (degrees C)",fontsize="large")
title("Tart temperature above baseline",fontsize="large")
plot(t, temp, "k-", linewidth=4)
plot([0,0],[0,12],linewidth=0)
grid(True)
show()
```

Question (5)

(This question was on the final examination in 2012, and worth 13 marks. Expected working time for this question was about 13 minutes.)

Note: When answering this question, do not refer to any of the "Before class" content. You do not need to use Lethal rate or any other concept defined in that work. Simply answer the question as presented.)

In the first stage of the heating process described in Question (4), tarts were heated at a linear rate for the first 10 minutes. The following Python program plots the temperature of the tarts **above the baseline of 60 $^\circ$C** for the first 10 minutes, with the linear rate of heating shown as a dashed line. Assume that during the first 10 minutes the temperature instead follows the solid curve shown in the graph. Run the Python program now, then answer these questions using the graph:

1. Find the equation of the new temperature from $t = 0$ to $t = 10$ mins. (Hint: during this time the graph forms one quarter of a cycle of a sine wave with no phase shift.) **Note**: you could "cheat" this question by reading the program. However, that misses the point of the question, which is for you to try to find the equation by examining the graph. The program was not on the exam paper.

2. **From the graph**, roughly estimate the area **between** the solid curve and the dashed line. (Hint: each grid cell in the graph has an area of 4.)

3. Using integration, it can be shown that the AUC of the first quarter of a cycle of a sine wave with period P and amplitude A is equal to $\dfrac{AP}{2\pi}$. Use this to find the percentage error in your answer to Part 2.

In [ ]:
```
# Program to plot the temperature profile of an egg tart during the initial he
ating phase

from pylab import *

t=array([0,10])
temp=array([10,10])
xlabel("Time (mins)",fontsize="large")
ylabel("Temperature (degrees C)",fontsize="large")
title("Tart temperature above baseline: initial heating",fontsize="large")
plot(t, t, "k--", linewidth=4)

t1=arange(0,10.05,0.1)
temp1=10*sin(2*pi*t1/40)
plot(t1, temp1, "k-", linewidth=4)
plot([0,0],[0,12],linewidth=0)
grid(True)
show()
```

Question (6)

This question is a required, in-class assessment piece. To receive the marks for this component, you and your partner must show your answers to a tutor during your tutorial.

As noted in Question (3), to help ensure that pork is safe for consumption, the United States Department of Agriculture published the following table of times for which the internal temperature should be maintained at the given levels.

Temp. ($^\circ$F)	Time (mins)		Temp. ($^\circ$F)	Time (mins)
120	1260		132	15
122	570		134	6
124	270		136	3
126	120		138	2
128	60		140	1
130	30		142	1

Write a program that plots the data from that table, and uses the trapezoid rule to calculate the area under that graph. You may paste the following skeleton program into the Python cell if you wish. Your program must define and use the function `trapArea`, which returns the area of a trapezium given the two heights and the width. Comment your program appropriately, then show it to your tutor.

```
from pylab import *

def trapArea(y1,y2,wid):
# Need to write this; your program MUST use this.


# Main program
temp = arange(120,144,2)
Time = array([1260,570,270,120,60,30,15,6,3,2,1,1])
numTemp = 12
.
.
.
while i < numTemp-1:
    .
.
show()
```

In [ ]:
```
# Paste and write your program here
```

Question (7)

(Parts 1 to 3 of this question were on the final examination in 2011, and worth 17 marks. Expected working time was about 17 minutes.)

Run the following Python program, which plots a graph showing the rate of power consumption by a household at any time t between 0 and 12 hours, with some values shown in the table. (Note that at time $t = 8$, there is a sudden increase in power consumption from 800 W to 1000 W; this is denoted "800; 100'" in the table.)

Time (hrs)	**0**	**2**	**8**	**10**	**12**
Power (watts)	1000	800	800; 1000	1000	1400

1. Let C represent the area under the power consumption curve. Find the **exact** value of C, and include units in your answer.

2. **By hand**, find all of the output produced by the following partial Python program which is intended to find the area C under the power consumption curve.

```
t = array([0, 2, 8, 10, 12])
p = array([1000, 800, 800, 1000, 1400])
tA = 0
a = zeros(5)
i = 0
while i<4:
    a[i] = p[i] * (t[i+1] - t[i])
    tA = tA + a[i]
    print("a[i] = ",a[i])
    i = i + 1
print("AUC = ", tA, "units. ")
```

3. The above Python code does not produce the correct value for the area under the curve. Briefly explain the error(s) in the program.

4. (This question was not on the exam paper.) Modify the above program it so that the output is correct. Check the calculated value of the AUC against the answer you obtained by hand in Part 1.

In [ ]:
```
# Plot graph showing the rate at which a house consumes electric power
from pylab import *

use=array([1000,800,800,1000,1000,1000,1400])
t=array([0,2,8,8,10,10,12])
plot(t,use,'k-',linewidth=3)
grid(True)
xlabel("Time (hours)",fontsize=20)
ylabel("Power (watts)",fontsize=20)
title("Rate of power usage",fontsize=20)
plot([0,1],[0,1500],'k-',linewidth=0)
show()
```

Question (8)

(This question was on the final examination in 2012, and worth 8 marks. Expected working time was about 8 minutes.)

The following text is the abstract of a research paper published in the journal *Diabetes Control* in 1994. Briefly interpret (so-called) "Tai's model" (http://care.diabetesjournals.org/content/17/2/152), and critically evaluate the quantitative and mathematical claims in the quote.

> **A Mathematical Model for the Determination of Total Area Under Glucose Tolerance and Other Metabolic Curves**</center>
> **Author: Mary Tai**
>
> OBJECTIVE: To develop a mathematical model for the determination of total areas under curves from various metabolic studies.
>
> RESEARCH DESIGN AND METHODS: In Tai's Model, the total area under a curve is computed by dividing the area under the curve between two designated values on the X-axis (abscissas) into small segments (rectangles and triangles) whose areas can be accurately calculated from their respective geometrical formulas. The total sum of these individual areas thus represents the total area under the curve. Validity of the model is established by comparing total areas obtained from this model to these same areas obtained from graphic method (less than ± 0.4 %). Other formulas widely applied by researchers under- or overestimated total area under a metabolic curve by a great margin.
>
> RESULTS: Tai's model proves to be able to 1) determine total area under a curve with precision; 2) calculate area with varied shapes that may or may not intercept on one or both X/Y axes; 3) estimate total area under a curve plotted against varied time intervals (abscissas), whereas other formulas only allow the same time interval; and 4) compare total areas of metabolic curves produced by different studies.

Extra questions

Here are some extra practise questions, for you to do in class (if you have time), or outside class. You do not need to do them all, but may like to choose some to help with your preparation for the final exam.

Question (9)

(This question was on the final examination in 2013, and worth 18 marks. Expected working time for this question was about 18 minutes.)

The following information about the African country Niger is useful for answering this question (figures for years after 2010 are predictions).

Year	**2010**	**2011**	**2015**	**2017**	**2020**
Total Nigerien population (millions)	15.6	16.1	18.5	19.9	22.1

1. Assume that the function $e(t) = 45 + t^2$ models predicted annual per capita electricity consumption in Niger until the year 2020, in units of kWh per person, where t is the number of years since 2010. Sketch a rough graph of $e(t)$, marking appropriate points.

2. Use the function $e(t)$ from Part 1 and the population predictions for Niger to calculate the predicted **total** annual electricity consumption in Niger in the years 2010, 2011, 2015, 2017 and 2020.

3. Plot the total annual electricity consumption values from Part 2 on a set of axes. Include a title, appropriate values, and axis labels and units. Join consecutive pairs of points with a straight line.

4. Find the units of the **gradient** of the graph your produced in Part 3, and find the units of the **area under the curve** (AUC) of the graph.

5. Calculate the **exact** AUC of the graph produced in Part 3. Show all working.

Question (10)

(This question was on the final examination in 2010, and worth 9 marks. Expected working time for this question was about 9 minutes.)

1. In class we saw that areas under curves can be calculated by using the Fundamental Theorem of Calculus and by using areas of rectangles. Describe some strengths and weaknesses of each approach (there is no need to describe the approaches). In particular, under what practical circumstances would each approach be used?

2. The concentration of a drug in an individual's blood is measured over a period of two hours, giving the values in the table. Use areas of rectangles to estimate the area under the curve when the drug concentration is plotted against time, and briefly explain the physical meaning of the area.

Measured concentration (mmol L^{-1})	**0**	**2**	**4**	**3**	**0**	**2**	**2**
Time (hours)	0	0.1	0.5	1.2	1.4	1.7	2.0