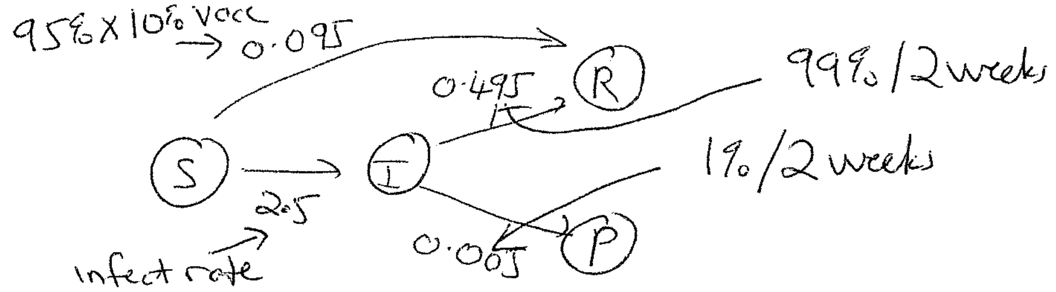
SCIE1000, Solutions to Tutorial Week 13.

(1) The next two pages contain hand-written answers which would receive full marks on an exam. (Pay careful attention to these solutions; we didn't hand-write them because we were too lazy to typeset. Instead, the solutions should give you a good idea of how little you need to write, how 'messy' you can be, and how you can use abbreviations.)

2014 exam. Question 6

1.



$$S' = -0.095S - 2.5\frac{IS}{N}$$

$$I' = 2.5\frac{IS}{N} - 0.495I - 0.005I$$

$$R' = 0.095S + 0.495I$$

$$P' = 0.005I.$$

2\. Conservation of people: no births or deaths.

3\. $S' = 90498 - 100000 = -9503$

$I' = 3 - 1 = 2.$

$R' = 9500 - 0 = 9500.$ $P' = 0.$

4\. Starts slowly; by Week 7 205 infected but in next 8 weeks this grows to 3105 infected. After 7 weeks 1 paralysed, rises to 138 by week 30. Most P cases occur after week 15 or so. Epidemic peaks around Week 15. Mostly over by Week 30. Most R people were vaccinated.

2014 exam. Question 6 continued

5. Because only 1% of I becomes P, and I is very low for some weeks at start, as the epidemic "takes off"

6. S' always ≤ 0 : people only leave S ($\rightarrow$ R or $\rightarrow$ I) and never return to S.

I' can be +ve and -ve : makes sense that numbers of I $\uparrow$ and $\downarrow$ (see table, eg).

R' : always ≥ 0 once removed/recovered, never leave that category

P' always ≥ 0 : Paralysis is permanent in this model.

(2) Answers will vary.

(3) Answers will vary.

(4) Answers will vary.

(5) 1. Each arrow that does not point to another circle indicates a transition to death. These transition probabilities are calculated by dividing the total number of deaths in each stage by the total population in that stage:

$$\frac{122}{7600} = 0.016 \quad \frac{30}{6600} = 0.005 \quad \frac{44}{1400} = 0.031.$$

As we are assuming that each person in the youth stage lives for 15 years, in one year one fifteenth of them will transition into adults. Thus the transition from youth to adult is $\frac{1}{15} = 0.067$. Similarly, the transition from adult to older person is $\frac{1}{35} = 0.029$.
Finally, in 2010, 6.6 million Nigeriens produced 700 thousand live births. This is $\frac{700}{6600} = 0.106$ births per adult. This gives the adults to youth transition probability.

2. From the table, we have that $Y_0 = 7600$, $A_0 = 6600$ and $O_0 = 1400$ thousand people. The initial rate of change of these values can be found by substituting into the differential equations:

$$\begin{aligned} Y_0' &= -0.083 \times 7600 + 0.106 \times 6600 = 68.8 \\ A_0' &= 0.067 \times 7600 - 0.034 \times 6600 = 284.8 \\ O_0' &= 0.029 \times 6600 - 0.031 \times 1400 = 148 \end{aligned}$$

Using Euler's method, we can then calculate

$$\begin{aligned} Y_1 &= Y_0 + \Delta t Y_0' = 7600 + 2 \times 68.8 = 7737.6 \\ A_1 &= A_0 + \Delta t A_0' = 6600 + 2 \times 284.8 = 7169.6 \\ O_1 &= O_0 + \Delta t O_0' = 1400 + 2 \times 148 = 1696 \end{aligned}$$

3. The prediction from Euler's method would be greater than the UN's prediction. The Euler's method prediction assumes that all population growth and death rates will stay the same, however the UN's prediction would likely take into account a decreasing population growth rate and death rates.

4. A. A very successful campaign promoting contraception would decrease the fertility rate of the Nigerien population, which would cause d to decrease.

   B. The new disease will (hopefully) not affect the youth population, and will probably not have a big effect on the older population, but should cause a higher death rate in the adult population. Also, if more sexual encounters result in death, rather than a child, the fertility rate should drop also. Thus, c will increase, f may increase a small amount, and d will likely decrease.

   C. Improvements in public health and nutrition should decrease the death rates of all parts of the population. So a, c and f should all decrease.

There will be some minor additional secondary effects, but the above show the main impacts. Notice that none of the scenarios affected the rates b and e. This is because they only depend on the number of year ages in each stage of the life cycle diagram, not the demographics of the Nigerien population.

(6) This was a task to do, not a question

(7) Here is an example of a hand-written answer which would receive full marks on an exam.

2014 exam Question 2.

1. Number infected $= 0.612 \times 500 = 306$

$\Rightarrow$ not infected $= 194$

Sensitivity $= 0.92$ specificity $= 90\%$

Sens $= \frac{A}{A+C}$ so $0.92 = \frac{A}{306}$ so $A \approx 282$

so $C = 306 - 282 = 24$

Spec $= \frac{D}{B+D}$ so $0.9 = \frac{D}{194}$ so $D = 175$ and $B = 19$

2. $A+B = 282+19 = 301$. prob(true +ve) $= \frac{A}{A+B} = \frac{282}{301} = 93.7\%$

3. Almost no-one in the global pop^n has polio, whereas 61.2% of the samples "had polio". Thus, the person's test is almost certainly a false +ve result

(8) Here is an example of a hand-written answer which would receive full marks on an exam.

2014 exam Question 4.

1. $AUC = 30235 + 2\times\left(\frac{~~3283~~35251+23484}{2}\right) + 3\times\left(\frac{23484+10487}{2}\right) + 4\times\left(\frac{5185+10487}{2}\right)$

$= 30235 + 58735 + 50956.5 + 31344$

$= 171270.5$

2. $\frac{178300 - 171270.5}{171270.5} \times 100\% = 4.1\%$

3. $\int_{25}^{\infty} P(t)\,dt = \left[-176000\, e^{-0.2t}\right]_{25}^{\infty}$

$= -176000 \times 0 - -176000 \times e^{-5}$

$\approx \underline{\underline{1186}}$. The AUC from $t=25$ to $t=\infty$ is 1186, so 1186 more cases.

4. It is not likely to be exact, but could be a good model. Remember, P(t) is just a predictive model. There is no guarantee it will happen. Local outbreaks in isolated populations may boost the number of cases.

(9) Here is an example of a hand-written answer which would receive full marks on an exam.

2014 exam Question 5.

1. Equil'm: 0.7

Amp: 0.8

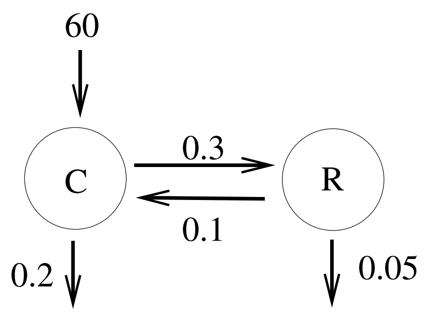
phase shift: 22 left

$L(t) = 0.7 + 0.8\sin\left(\frac{2\pi P}{52}(t-22)\right)$.

2. Smallest: about Week 10: $10^{-0.75} \approx 0.18$

largest: about week 30: $10^{1.55} \approx 35.5$

(10) 1. The life-cycle diagram is:



2. Consider the rate of change of each population size.

   For C', each month the number rises by the 60 new cases and the 10% of people in remission who have their cancer redevelop, and reduces by 50%, being the 20% who die and the 30% who move into remission.

   For R', each month the number rises by the 30% of people with cancer who go into remission, and reduces by 15%, being the 10% who have their cancer redevelop and the 5% who die.

3. When $C = 300$ and $R = 40$, we have:

   $C' = 60 - 0.5 \times 300 + 0.1 \times 40 = 60 - 150 + 4 = -86$ and

   $R' = 0.3 \times 300 - 0.15 \times 40 = 90 - 6 = 84$.

   Hence using a step size of 0.5 gives:

   $C = 300 - 0.5 \times 86 = 257$ and $R = 40 + 0.5 \times 84 = 82$.

4. A. It appears that the values of R and C are likely to approach constant values. That is, there will be 200 people with cancer and 400 in remission.

   B. The approach is OK in the short term, as R resembles a power function over the given time period. However, in the long term, it appears that R may not or will not exceed 400. The power function will increase indefinitely (at a decreasing rate, but still indefinite). Hence the model will be poor over an extended time period.

5. The output from the program is:

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T= 0.0 : C' = -86.0  and R' = 84.0
            C = 257.0 and R = 82.0
T= 0.5 : C' = -60.3  and R' = 64.8
            C = 226.85 and R = 114.4
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