SCIE1000, Solutions to Tutorial Week 3.

(1) The volume and proportion of water of most bodily tissues such as skin, bone and organ tissue does not vary much between individuals. The percentage of body fat varies much more, and body fat does not contain as much water by volume as other tissues, so this is the factor we will consider when answering these questions.

1. If two people have the same height and gender, but different masses, the one with more mass will generally have a larger percentage body fat, and thus a lower body water proportion.
2. If two people have the same mass and gender, but different heights, the taller one will be generally skinnier than the shorter one. This implies the shorter person has the larger percentage of body fat, and thus the lower body water proportion.
3. Females typically have a higher percentage of fat than males. If two people had the same mass and height, but were different genders, the the female would typically have the lower body water proportion.

(2) From Question (1), we have estimated that mass is roughly inversely proportional to BWP and height is roughly proportional to BWP, so our model should look roughly like $BWP \approx a\frac{H}{M} + c$ for some unknown constants a and c. The value a maybe different for males and females. Say it is a for males and b for females. We can incorporate this into our equation like so:

$$BWP \approx (a(1-G) + bG)\frac{H}{M} + c.$$

(3) A conditional allows a program to behave differently depending on whether something is True or False. Thus, different input vales can lead to different outcomes. A conditional statement is used when decisions need to be made in the program, often based on whether the value of a certain variable is greater than, less than or equal to a particular value. Conditional statements are extremely important in a program which needs to perform different calculations depending on the input given, which is almost all programs.

(4) Here is the output:

```
Enter mass in kg: 45
Enter BWP: 0.64
How much pure alcohol consumed in g: 20
Peak Blood Alcohol content =  0.0694444444444
```

(5)
1. The following lines should be included at the end of the program.

```
if BACpercent == 0:
    print("Zero alcohol")
elif BACpercent < 0.05:
    print("Legal to drive on an open licence")
else:
    print("Not legal to drive")
```

2. We find the amount of alcohol consumed a that makes $B = 0.05$. First, the amount of water in the person is $0.64 \times 45 \times 1000 = 28800$ g.
   Then $100a/28800 = 0.05$, so $a = 14.4$ g.

(6) Discussion.

(7) Discussion.

(8)
1. $M = 38.5$ kg, and $H = 1.55$ m. In all of the following answers, the value of r is a proportion or fraction. Then:

$$\begin{aligned} r_f &= 0.31223 - 0.006446 \times 38.5 + 0.4466 \times 1.55 \\ &= 0.756 \end{aligned}$$

2. $M = 107$ kg, and $H = 1.88$ m. Therefore:

$$\begin{aligned} r_m &= 0.3161 - 0.004831 \times 107 + 0.4632 \times 1.88 \\ &= 0.670 \end{aligned}$$

3. Substituting a fixed mass of 70 kg into each formula gives:
   Females: $r = -0.13899 + 0.4466H$.
   Males: $r = -0.02207 + 0.4632H$.
   See Figure 1 for the graphs.

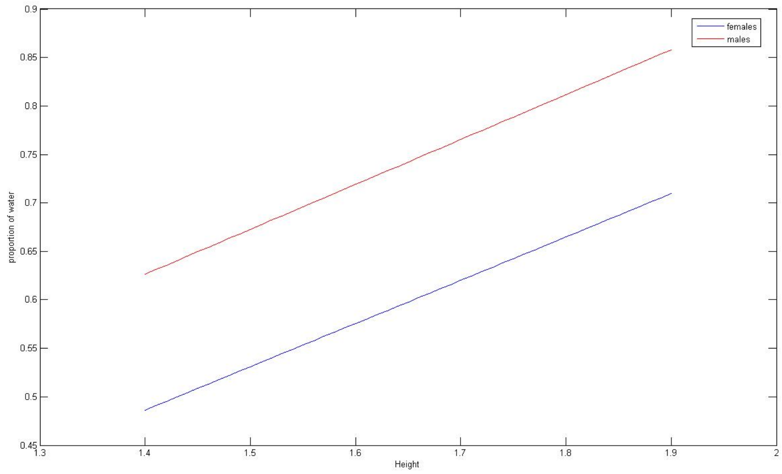


Figure 1: *A graph of the estimate for the proportion of water for males and females, for a fixed mass of 70 kg, for heights between 1.4 m and 1.9 m.*

4. Given the fundamental anatomical differences between males and females (for example, additional breast tissue in females), the graph for males must be shifted vertically upwards compared to that for females, so must have a larger y-intercept. The slopes of both graphs are fairly similar, although that for males is lightly larger. This suggests that, for a fixed mass, males add slightly more lean tissue for increased heights than do females.
5. Here is one possible solution.

```
# A program to calculate values of Widmark's r.
from pylab import *

mass = eval(input("What is the mass (in kg)? "))
heightM = eval(input("What is the height (in m)? "))
gender = eval(input("Enter 0 for male, anything else for female: "))

if gender == 0:
    # Estimate Widmark r for males
    r = 0.3161 - 0.004831*mass + 0.4632*heighttM
else:
    # Estimate Widmark r for females
    r = 0.31223 - 0.006446*mass + 0.4466*heightM

print("The value of Widmark's r for this person is: ", r)
```

6. Here is the output from running the program twice. In both cases, the output matches the values calculated previously.

```
What is the mass (in kg)? 107
What is the height (in m)? 1.88
```

```
Enter 0 for male, anything else for female: 5
The value of Widmark's r for this person is:  0.756289

What is the mass (in kg)? 38.5
What is the height (in m)? 1.55
Enter 0 for male, anything else for female: 0
The value of Widmark's r for this person is:  0.669999
```

(9) Here is one possible solution.

```
# A program to calculate values of Widmark's r.
from pylab import *

print("This program calculates Widmark's r value.")
mass = eval(input("What is the mass (in kg)? "))
heightM = eval(input("What is the height (in m)? "))
gender = eval(input("Enter 0 for male, anything else for female: "))

if gender == 0:
    # Estimate Widmark r for males
    r = 0.3161 - 0.004831*mass + 0.4632*heightM
else:
    # Estimate Widmark r for females
    r = 0.31223 - 0.006446*mass + 0.4466*heightM

# Estimate the total mass of water
TotalMassWater=r*mass

print("The value of Widmark's r for this person is: ", r)
print("The total mass of water in this person's body is: ", TotalMassWater)
```

Using the program, the 1920s average woman contained 38.406 kg water, whereas the modern woman contains 41.359 kg. Only 2.953 kg of the 12 kg increase in mass for women is water, which means 9.047 kg is non-water. Also using the program, the 1920's average man contained 55.745 kg water, where as the modern man contains 61.653 kg. Only 5.908 kg of the 13 kg increase in mass for men is water, and 7.092 kg is non-water. As most of these increases are non-water, it is likely that the increase in mass is due to an increase in body fat, which contains less water relative to other body tissues. This agrees with other known trends.

(10) This question is mostly calculation based.

1. Let $B(t)$ be the person's BAC in $\%$ of blood volume at any time t in hours after some time 0. Recall that the equation of a straight line as a function of t is $y = mt + c$, where m is the gradient or rate of change.
   The value of m is -0.03% in 2 hours, so the rate of change is $-0.015\%/hr$ (and the negative sign is important, as the BAC is dropping).
2. Since the rate of change is roughly constant, the total change will be the total time times the rate of change, unless all the alcohol is eliminated before the final time. The time we are interested in is 3.5 hours long, and from Part 1, the rate of change is $-0.015\%/hr$, so the total change will be
   $$-0.015 * 3.5 = -0.0525\%.$$
   The magnitude of this change is less than the 0.12% present at 9 pm, which means it is possible for this much alcohol to be eliminated (and the negative sign is important, as the BAC is dropping).
3. Let's model BAC over time with a linear equation. From Part 1, $B(t) = -0.015t + c$. Let $t = 1$ at 9pm, and substitute $t = 1$ and $B(1) = 0.12$ into the equation for $B(t)$, so $0.12 = -0.015 \times 1 + c$, so $c = 0.135$. Hence $B(t) = -0.015t + 0.135$. Now we want to know the time $((t)$ when $B(t) = 0.05$. Subbing this into our equation and rearranging for t gives $t = \frac{B(t)-0.135}{-0.015} = 5.67hr$. 5 and 2/3 of an hour after 8pm is 1:40am. This is the time the BAC drops below 0.05%.

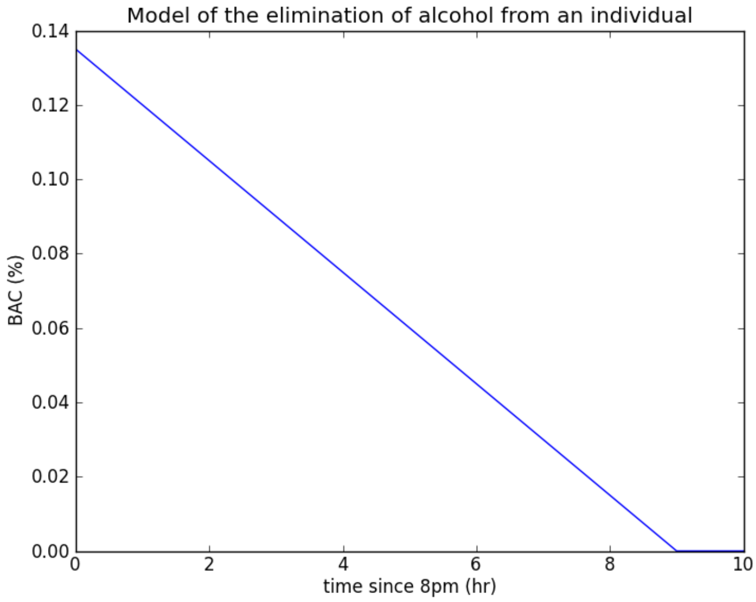


Figure 2: *A graph of the elimination of alcohol from an individual from 8 pm until 6 am.*

4. See Figure 2.

(11)

1. For ease of notation, let M represent the mass of the person, in the identified units. There are 2.2 pounds per kg, so

$$M \text{ pounds } = M \text{ g } \times \frac{1}{1000} \text{ kg g}^{-1} \times 2.2 \text{ pounds kg}^{-1}.$$

Let A be the amount of alcohol consumed. Each drink contains 10 g of alcohol, so

$$n \text{ drinks } = A \text{ g } \times \frac{1}{10} \text{ drink g}^{-1}.$$

Substituting these into the original equation, we have

$$\begin{aligned} t &= \frac{240n}{M} \\ &= \frac{240 \times A \text{ g } \times \frac{1}{10} \text{ drink g}^{-1}}{M \text{ g } \times \frac{1}{1000} \text{ kg g}^{-1} \times 2.2 \text{ pounds kg}^{-1}} \\ &= \frac{10909 A \text{ g}}{M \text{ g}}. \end{aligned}$$

Therefore, $t = 10909A/M$ is an estimate of the time it takes for the BAC of a person weighing M g to return to zero after consuming A g of alcohol.

2. Rearranging the Widmark formula for t, we obtain:

$$t = \frac{100A}{0.015rM} = \frac{6667A}{rM} \tag{1}$$

3. Substituting $r = 0.67$ (for this man) into the equation in Part 2, we have:

$$t = \frac{9950A}{M} \tag{2}$$

Using $A = 40$ g and $M = 107000$ g, we have the following estimates:

$$\begin{aligned}\text{Part 1: } t &= \frac{10909A}{M} = 4.08 \text{ hrs}\\ \text{Part 2: } t &= \frac{9950A}{M} = 3.72 \text{ hrs}\end{aligned}$$

4. Comparing our results for Part 3, we can see that the two equations are fairly similar; they only differ in the value of the constant. The equation from Part 1 (based on an alcohol consumption guide published by the US government) estimates a time that is about 10% longer than the equation from Part 2 (based on the Widmark formula). This makes sense: the published guide <u>should</u> be more conservative.

(12) Make sure you think about this before starting, so you have a good approach to use. We first need to estimate the volume of the human biomass; to do this, you need to estimate the population. As long as you state what you have assumed, and the assumption is not clearly silly, then you will receive the marks.

1. Population: 7 billion, or 7×10^9. Average mass per person: 70 kg.
   Then total mass $= 7 \times 10^9 \times 70 \approx 5 \times 10^{11}$ kg.
   There are 1000 kg per m^3, so the volume is $5 \times 10^{11} \div 1000 = 5 \times 10^8$ m^3.
   Then the volume of a cube of side s is s^3, so $s^3 = 5 \times 10^8$, so $s \approx 800$ m.
2. The expression is $s = \left(\frac{70N}{1000}\right)^{1/3}$.
3. The expression for s is a power function, with power between 0 and 1. This is the same shape as a species area curve, so the graph is increasing, but at a decreasing rate. You need to sketch the graph.

(13) This is the second part of an exam question; the first part was given on the tutorial sheet last week. Overall, the question was very poorly done on the exam. Take time to think about the answer first. You are given the equation for M. For a fixed length, the equation is a quadratic in G; for a fixed G, the equation is linear in L. This tells you the basic shapes of the graphs.

The next page shows an example of a hand-written answer that would receive full marks on an exam. (Pay careful attention to the solution; we didn't hand-write it because we were too lazy to type the answer. Instead, it should give you a good idea of how little you need to write, how 'messy' you can be, and how you can use abbreviations.)

(14)
1. We have that S(25) $= 10 \times (25)^{0.5} = 50$ and S(100) $= 10 \times (100)^{0.5} = 100$ Therefore:

$$\begin{aligned}\frac{\Delta S}{\Delta a} &= \frac{100-50}{100-25}\\ &= \frac{50}{75} = \frac{2}{3}\end{aligned}$$

   So the average rate of change is $2/3$ of a species for every square kilometre increase in size.
2. **(Note: many possible reasons are acceptable why the two species-area curves differ. Any reasonable and justified argument will be awarded marks.)**
   Some reasons could include:
   - S_2 could have a wider variety of of landscapes within its boundaries (e.g. dense forest, open plain, a lake etc) which would explain the higher growth rate.
   - S_1 may have been partially cleared in certain areas to make way for footpaths, roads, picnic tables or landscaping etc, this would reduce the amount of biodiversity.

(15) 1.A. Athens

1.B. Rome

1.C. Munich

1.D. Paris

1.E. London

1.F. Moscow

1.G. Munich

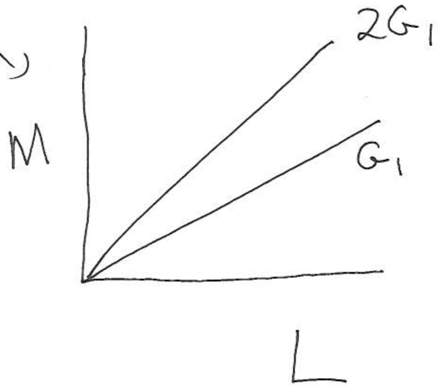
2.A. This will occur for any values which satisfy the inequalities $\mathrm{v1} \leq 4$, $\mathrm{v2} < 0$ and $0 < \mathrm{v1} + \mathrm{v2} < 4$. An example of such a pair would be $v1 = 3$ and $v2 = -1$.

2.B. This will occur if $4 < \mathrm{v1} = v2 <= 6$, i.e. if $\mathrm{v1} = 5$ and $\mathrm{v2} = 5$.

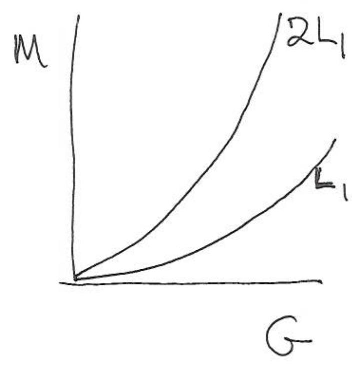
2.C. This will occur if $\mathrm{v1} = \mathrm{v2} > 6$, i.e. if $\mathrm{v1} = 7$ and $\mathrm{v2} = 7$.

Handwritten answer to the earlier question.

2(a)



(b)



(c) P_1: $M = 79.6\, G_1^2 L_1$

P_2: $M = 79.6\, G_2^2 L_2$.

Now $r_1 = 1.2 r_2$, so $G_1 = 1.2 G_2$, and both Ms are equal

Hence $\cancel{79.6}\,(1.2G_2)^2 L_1 = \cancel{79.6}\, G_2^2 L_2$.

so $1.44 L_1 = L_2$

so $\dfrac{L_1}{L_2} = \dfrac{1}{1.44} \approx 0.7$