SCIE1000 Tutorial sheet 5

This tutorial contributes toward your final grade; see the Course Profile (https://www.uq.edu.au/study/course.html?course_code=SCIE1000). The tutorial will be marked out of 6, with 3 marks for completing the "Before class" work, and 3 marks for completing the "In class" assessment and working on the remaining "In class" questions until you finish them or the tutorial ends.

Goals: This week you will work through some general calculation and discussion questions, mostly relating to population growth and exponential functions. As usual, the broad concepts and techniques are more important than the specific examples. Do not try to commit lots of facts to memory; instead, know **how** to do things, and **when** certain models and approaches are appropriate.

We do not cover any new Python content this week. You can take this opportunity to consolidate your understanding of what we have already covered, including variables, printing, input, conditionals and loops. Ask your tutor if you are confused about anything.

To be completed before class

Complete the following questions before class, write (or type if you wish) your answers on a sheet of paper, put your name and student number on the top of the paper, and hand it to your tutor as you enter the room. **If you do not hand in the answers at the start of the class, as you enter the room, then you will lose the marks for this component.** Note that in some cases there are no "right" or "wrong" answers.

Question (1)

This question involves watching a video that runs for more than one hour. Give yourself plenty of time!

Professor Al Bartlett (https://en.wikipedia.org/wiki/Albert_Allen_Bartlett) was an Emeritus Professor in Physics at the University of Colorado at Boulder, who died in 2013. He presented a lecture called "Arithmetic, Population and Energy" more than 1700 times. A copy of this lecture, split into eight parts, can be found here (https://www.youtube.com/watch?v=sI1C9DyIi_8). This video was recorded in 2002, so some aspects are a bit dated. For example, 2002 was before global warming was widely recognised as a problem, and technology has enabled extraction of oil that previously was not possible. However, the core messages are still absolutely relevant, and the video gives an excellent intuitive "feel" for some properties of exponential functions.

Watch the entire video - truly, it's more interesting than you might expect - **and take some useful notes as you watch it**. (There may be questions relating to this on the final exam.) You do not need to note down details about Colorado or the USA, or specific facts about energy usage. Instead, answer the following:

1. What are the general principles and key points that Professor Bartlett makes about growth in populations and resource usage, properties of exponential functions, and how well exponential functions are understood in general?

2. Did you find anything about the video particularly interesting? What new, broad ideas did you learn, if any? How did the video make you **feel**, and what did it make you **think**? Come along to class ready to discuss the answers to these questions. The notes you have taken will be the evidence that you can show your tutors to verify that you have done this task.

3. Early in the video, Professor Bartlett discussed the doubling time for the price of a one day ski lift pass in Vail, Colorado. In the 2016-17 ski season, if you bought your ticket at the resort, the price was $179 (US dollars). Comment on this price in relation to what Profssor Bartlett predicted in 2002.

To be completed in class

Complete the following questions in class. They involve a mix of individual work, and discussions with others. Make sure that you read the questions before class and think about how you might approach answering them. Don't rely on someone else doing all of the work. You need to work by yourself on the final exam, so it is important that you work hard now.

Feedback: Be proactive!

Australian government research shows that students often feel they don't receive adequate feedback on their work. In a class of 800 students, it is not possible for the course coordinator to give direct feedback to each student. Instead, tutorial classes are designed to be the place in which you can get feedback on your work from classmates and the tutors. You can ask for help, show them your answers, and discuss your understanding of any of the course material. As an adult learner, the onus is on **you** to seek feedback; tutors and classmates are happy to give it, if you want it.

Question (2)

Your tutors will record the marks for the sheet of paper you submitted with the notes you took while watching the video before class, and they will then return your sheet to you.

1. Discuss your notes with your partner. If they have written something that you think may be important and that you missed, then update your notes.

2. Discuss the content of the video as a group, and update your notes appropriately.

Question (3)

The *World Fact Book* (https://www.cia.gov/library/publications/the-world-factbook) gives various global and country-based demographic, economic, geographic and other data. Use this *World Fact Book* link (https://www.cia.gov/library/publications/the-world-factbook/geos/xx.html) to find the global *proven oil reserves*, and the daily consumption of refined petroleum products. (For simplicity, we will assume that one barrel of crude oil results in the production of one barrel of refined petroleum products.)

1. If no more oil is discovered and daily consumption remains unchanged, when will global oil reserves run out?

2. Is your answer to Part 1 likely to occur in reality? If not, what would you expect to happen, and why? Discuss the answer.

3. One prediction for total oil consumption in the year 2030 is 37,960 MBPY. Assuming that oil consumption follows an exponential function with a constant growth rate, find a function that models annual oil consumption between the year for which the latest data are given in the *World Fact Book* and 2030 as a function of the year t, where $t = 0$ in the year in the *World Fact Book*.

4. On the video you watched in Question (1), it was stated that the area under the oil consumption curve represents **total** consumption of oil in a given time period. As you might recall from school, this area can be found by integrating the consumption function. Let Q be the total quantity of proven oil reserves, with current consumption C per year, an exponential growth rate in consumption of k per year, and T be the time at which the oil reserves will run out. Then integrating the function in Part 3 gives:

   $$Q = \frac{C}{k}e^{kT} - \frac{C}{k}.$$

   Rearrange this formula to give an expression for the time T at which oil will run out.

5. Assuming that your model in Part 3 is correct and that no more oil is discovered, use your answer to Part 4 to find when global oil reserves will run out.

6. Compare your answers to Parts 1 and 5, and comment on the values.

Question (4)

The video in Question (1) mentions the *Rule of 70*, which allows the doubling time of exponentially increasing quantities to be estimated easily without a calculator. The rule says that if the exponential growth rate *expressed as a percentage* equals r, then the approximate doubling time can be found by dividing 70 by r.

The *Wikipedia* entry (https://en.wikipedia.org/wiki/Rule_of_72) for the *Rule of 72* also mentions the *Rule of 70* and *Rule of 69*, all of which are applied in a similar way. When estimating a doubling time, the value to be used (69, 70 or 72) is chosen so as to make the division simplest: for example, for a growth rate of 6 %, 72 is easiest (because $72/6 = 12$), whereas for a rate of 5 %, 70 is easiest (because $70/5 = 14$). The value of 69 is most accurate, but less convenient for dividing (unless the growth rate is 3 % or 23 %).

Justify (mathematically) the *Rule of 72* (or 70, or 69). (Hint: find the doubling time for a quantity with growth rate k, and note that, expressed as a percentage, the growth rate is $r = 100k$.)

Question (5)

Use the *World Fact Book* (https://www.cia.gov/library/publications/the-world-factbook) to find Australia's total population, nett migration rate, birth rate and death rate. Use these data to answer the following questions.

1. What is the current annual percentage growth rate of the Australian population?

2. Assuming the population is growing exponentially, and that the exponential growth rate matches your value from Part 1, write a mathematical function that models Australia's population at any time t years from now.

3. Use the *Rule of 72* to estimate the doubling time for the Australian population. Mathematically calculate the doubling time, and compare your answers.

4. Use your equation from Part 2 to write a program in the following Python cell to display Australia's predicted population each year for an inputted number of years. Use the program to check your answer to Part 3.

5. Data in the World Fact Book show that Japan has a substantially higher death rate than Australia, but people living in Japan also have a higher life expectancy than Australians. Explain briefly why these two facts are not inconsistent.

In [ ]:

```
# Write your program here
```

Question (6)

This question is a required, in-class assessment piece. To receive the marks for this component, you and your partner must show your answers to a tutor during your tutorial.

Much of Australia's current and recent prosperity, including the so-called "mining boom", is due to rapid growth in the Chinese economy. Use the *World Fact Book* (https://www.cia.gov/library/publications/the-world-factbook) to find China's total population and population growth rate, the total annual value of the Chinese economy (or *Gross Domestic Product*, GDP) in "purchasing power parity" US dollars, the GDP real growth rate and the GDP per capita.

In the Python cell below, write a program that calculates and prints the estimated Chinese population, GDP and GDP per capita, each year for the next 25 years. Your program should also print how many times larger the Chinese economy and Chinese GDP per capita is each year, compared to their initial values. Assume that both the population and GDP growth rates remain constant at their current values. (Hint: round population, GDP and GDP per capita output to zero decimal places, using the command `round(...)`.)

In [ ]:

```
# Write your program here
```

Question (7)

Typically, the rate at which an individual consumes resources (such as food, water and energy) and produces waste (such as CO_2 emissions, solid waste and so on) is proportional to their level of wealth, which in turn is often proportional to the average GDP per person in the population. Currently, average GDP per person in the USA is about 4 times the figure for China, and average energy consumption per person in the USA is about 3.5 times that in China.

Answer the following in the context of your answers to Question (6).

1. How many times larger than now is the Chinese economy predicted to be in 25 years?

2. How many times larger than now is Chinese GDP per person predicted to be in 25 years?

3. Assuming that the predictions are accurate:

   A. What do you predict that this will mean for the Keeling curve during the next 25 years?

   B. What do you predict that this will mean for the global availability and cost of resources, such as oil, coal, food and iron ore, during the next 25 years?

4. Do you think that the population and GDP predictions 25 years from now are realistic? If not, why not, and what do you think will happen instead? Why?

5. Do you think that people should be allowed to have as many children as they wish? Discuss points for and against your view.

Question (8)

(This question was on the final examination in 2013, and worth 13 marks. Expected working time for this question was about 13 minutes.)

1. Find an exponential model of the population of the country Niger between the years 2000, when the population was 10.9 million, and 2010, when the population was 15.6 million. (Hint: assume the growth rate was constant between 2000 and 2010.)

2. Use the *Rule of 72* to find the approximate doubling time of the Nigerien population.

3. Use your answer to Part 2 to predict the approximate Nigerien population in the year 2090, assuming the same exponential growth rate is maintained. (Hint: the elapsed time is 80 years.)

4. The UN prediction of the Nigerien population in 2090 is about 122 million. What physical factors could explain the difference between this figure and your answer to Part 3?

Extra questions

Here are some extra practise questions, for you to do in class (if you have time), or outside class. You do not need to do them all, but may like to choose some to help with your preparation for the final exam.

Question (9)

(This question was on the midsemester examination in 2011, and worth 7 marks. Expected working time for this question was about 7 minutes.)

Ecologists are considering two models for the biomass (in tonnes) of fish in a pond:

$$M_1 = 20(1 - e^{-0.2t}) \qquad \text{and} \qquad M_2 = 5\sqrt{t},$$

where t is the number of months since the most recent harvest.

1. One of the above models includes a maximum *carrying capacity* for the fish (that is, a maximum limit to the population size), and the other model increases indefinitely. Identify which of M_1 and M_2 has the carrying capacity, and explain your answer briefly.

2. Find the time at which $M_1 = 10$ and the time at which $M_2 = 10$.

Question (10)

(This question was on the final examination in 2008, and worth 4 marks. Expected working time for this question was about 4 minutes.)

Two species of algae have population sizes which are growing exponentially. Species A has an initial population of 1000 per mL, and Species B has an initial population of 3000 per mL. The growth rate of Species A is 3 % per hour, and the growth rate of Species B is 1 % per hour. At what time will the population sizes be equal?

Question (11)

(This question was on the final examination in 2010, and worth 11 marks. Expected working time for this question was about 11 minutes.)

A species of bacterium reproduces by individuals splitting into two. A population of this bacterium has a constant doubling time. At time $t = 0$ hours the population is 10^4 individuals, and at time $t = 5$ hours the population is 10^5 individuals.

1. Find the average rate of change of the population size between $t = 0$ and $t = 5$.
2. How many generations are there between $t = 0$ and $t = 5$? (Give your answer correct to one decimal place.)
3. Find the doubling time of this population.
4. At what time will there be 10^8 individuals in the population?

Question (12)

(This question was on the final examination in 2009, and worth 8 marks. Expected working time for this question was about 8 minutes.)

Recall that the equation for a species area curve is

$$S(a) = Ca^p,$$

where $S(a)$ is the number of species observed in an area of size a, and C and p are constants. An ecologist conducts a species-area survey of a habitat, and plots a graph with $\log_{10}$ of the observed number of species on the y axis and $\log_{10}$ of the area on the x axis.

1. The ecologist notices that the graph is linear. Show mathematically why this is, and interpret the physical meaning of the y-intercept and the slope of the graph.
2. Two of the points on the above graph are $(0, 1)$ and $(2, 2)$. Find the values of C and p in the equation $S(a) = Ca^p$ for this habitat.

Question (13)

(This question was on the final examination in 2009, and worth 8 marks. Expected working time for this question was about 8 minutes.)

1. The pH scale is a logarithmic scale, with a base of 10. At time 0 the pH of a solution is 8, and after 2 minutes the pH is 7. Find the **average rate of change of the concentration of hydrogen ions** in the solution over this time. Use units in your calculations.

2. If the concentration of hydrogen ions in the solution is changing at a linear rate, at what time will the solution have pH equal to 5?

Question (14)

(This question was on the deferred examination in 2010, and worth 11 marks. Expected working time for this question was about 11 minutes.)

A certain material undergoes radioactive decay. At time $t = 2$ hours a sample of the material contains 10^5 mg, and at time $t = 8$ hours the sample contains 10^4 mg.

1. Find the average rate of change of the sample size between $t = 2$ and $t = 8$.

2. Find the half life of this material.

3. At what time will there be 10^2 mg in the sample?

Question (15)

(This question was on the midsemester examination in 2012, and worth 8 marks. Expected working time for this question was about 8 minutes.)

When an object of initial temperature T_0 is moved to a location in which the temperature is a constant C, then the temperature T of the object at any time t is as follows, where k is a constant:

$$T = C - (C - T_0)e^{-kt}.$$

1. An object with initial temperature 60 $^\circ$C is placed in an oven with constant temperature 70 $^\circ$C, and a second object with initial temperature 80 $^\circ$C is placed in another oven with constant temperature 60 $^\circ$C. In both cases, $k = 0.1$ min^{-1}. Find the time at which both objects have the same temperature, and find that temperature.

2. A third object with initial temperature 100 $^\circ$C is placed in an oven with constant temperature 60 $^\circ$C. The value of k is unknown; however, after about 10 minutes the temperature of the object is 80 $^\circ$C. Hence estimate the time at which the temperature of the second object will equal 65 $^\circ$C, and explain your answer.

Question (16)

A very widely used method of estimating the number or quantity of something that is otherwise hard to measure is to take a sample, and then extrapolate that to give the required estimate.

1. A marine biologist, wanting to estimate the number of fish N that live on an isolated reef, captured a sample of S_1 individuals, tagged them and released them. One month later, she collected another sample of size S_2 and found S_3 tagged individuals amongst them. Assuming that the population has remained constant, develop a formula to estimate N from these values. Explain your answer. How accurate is this approach?

2. There are multiple ways of measuring the volume of blood inside a person. Many of these methods use a technique closely related to the fish-counting approach in Part 1. The following text is taken from the documentation lodged as part of *US Patent 5685302 - Method for determining plasma volume, determination of blood volume thereby, and apparatus therefore.* (https://www.google.com/patents/US5685302)

   Summarise briefly, in plain English, how the patent enables blood plasma volume to be measured.

> The subject invention provides a method for determining in a subject the volume of plasma in the subject's circulation which comprises:
>
> A. introducing into the subject's circulation a predetermined amount of a pharmaceutically-acceptable solution comprising a predetermined quantity of biodegradable, nontoxic macromolecules, which macromolecules are sufficiently larger than endothelial junctions in the subject's capillaries so that they are incapable of permeating the subject's capillaries, and each of which macromolecules is labeled with a detectable marker;
>
> B. allowing the solution to circulate for a period of time sufficient to distribute the macromolecules throughout the subject's circulatory system;
>
> C. obtaining a sample of plasma from the subject;
>
> D. determining the concentration of macromolecules in the sample by quantitatively measuring the detectable marker in the sample; and
>
> E. calculating the volume of liquid which would dilute the sample to the concentration determined in Step D from the predetermined amount of solution introduced into the subject's circulation and the predetermined quantity of macromolecules contained therein, thereby determining the volume of plasma in the subject's circulation.

3. Explain how the approaches in Parts 1 and 2 relate to each other.