

(1) 1. – **Number of Deaths**

The *number of deaths* is simply the total number of deaths in a time period that can be attributed to a particular cause. This gives a useful measure of the impact of each cause/risk factor in terms of total loss of lives.

– **Mortality rate**

The *mortality rate* is the number of deaths in a time period divided by the total relevant population. It is often specified as a rate per hundred, thousand or hundred thousand people.

– **Years of Potential Life Lost (YPLL or YLL)**

This is calculated by multiplying the number of deaths that occurred at each age by the standard life expectancy at that age. This gives a better measure of the *impact* of deaths; in other words, everyone has to die of something, at some age. YPLL weights premature or early death much more highly.

– **Years Lost Due to Disability (YLD)**

YLD for a particular cause in a particular time period are estimated as follows:

$$\text{YLD} = \text{number of incident cases in that period} \times \text{average duration of the disease} \times \text{disability weight.}$$

The disability weight reflects the severity of the disease on a scale from 0 (perfect health) to 1 (death).

– **Disability Adjusted Life Years (DALY)**

DALY is a method of measuring the impact of death and disability at different ages. One DALY can be thought of as one lost year of healthy life, and the burden of disease can be thought of as a measurement of the gap between current health status and an ideal situation where everyone lives to some standard life expectancy, free of disease and disability. DALYs for a disease or injury are calculated as the sum of the years of life lost due to premature mortality (YLL) in the population and the years lost due to disability (YLD) for incident cases of the disease or injury.

2. Notes on Key Figures and Graphs.

- Leading causes of death are high blood pressure and tobacco use. Leading factors in the global burden of disease are childhood underweight, unsafe sex and alcohol use. Three of these things (tobacco use, unsafe sex and alcohol use) could be prevented by changing behaviour, rather than medicine.
- High blood pressure alone accounts for 13% of deaths globally.
- Being over weight is responsible for 7% of deaths globally.
- People with different levels of income are more at risk from different diseases.
- The proportion of people with low income effected by disability compared to people with higher income is significantly greater than the proportion of people with low income who are effected by disease. In other words, people with high income experience much less of the total burden of disease, even less than their share of death due to disease.
- Burden of disease attributable to (lack of) contraception is significant in the developing world. However, percentage of deaths due to tobacco is more prevalent in the developed world.
- Environmental risks such as unsafe water, sanitation and indoor smoke from solid fuels cause around 2 million child deaths per year.
- Tobacco is a leading risk factor for mortality, responsible for 5.1 million deaths. Almost 1 in 8 deaths of people over the age of 30 is due to smoking.
- Unsafe sex, which leads to transmission of human papillomavirus, is responsible for virtually all deaths due to cervical cancer. Cervical cancer is responsible for 11% of global unsafe sex deaths and is the leading cause of cancer death in Africa.

Here are some important quotes from Part 4: Burden of disease: DALYs.

- Using DALYs, the burden of diseases that cause early death but little disability (eg. drowning or measles) can be compared to that of diseases that do not cause death but do cause disability (e.g. cataracts causing blindness).
- The global average burden of disease across all regions in 2004 was 237 DALYs per 1000 population, of which about 60% was due to premature death and 40% to non-fatal health outcomes.
- Almost one half of the disease burden in low- and middle-income countries is now from noncommunicable diseases.

- In low and middle income countries, children younger than 5 experience a 6 times greater proportion of the burden of disease than in higher income countries.
- Four primarily non-fatal conditions are among the 20 leading causes of burden of disease; these are unipolar depressive disorders, adult onset hearing loss, refractive errors and alcohol use disorders.
- Unipolar depression makes a large contribution to the burden of disease, being at third place world-wide and eighth place in low-income countries, but at first place in middle- and high-income countries. Effective treatments for depression are available, suggesting that this burden could be reduced.
- Mental disorders are an important source of lost years of healthy life for women aged 15-44 years.
- One sixth of the disease burden in adults is caused by injuries.
- The category of intentional injuries (including self inflicted injuries, suicide, violence and war) accounts for an increasing share of the burden, especially among economically productive young adults.
- Global DALYs are projected to decrease from 1.53 billion in 2004 to 1.36 billion in 2030, an overall decline of about 10%. Since the population increase is projected to be 25% over the same period, this represents a significant reduction in the global per capita burden.

(2) The `plot` command in Python can be used to plot graphs on the screen. The command requires at least two arrays of the same length as input. The first array contains the independent data, or the x coordinates of the points to be plotted, and the second array contains the dependent data, or the y coordinates of the points. In its simplest form, the command plots a graph with a solid blue line connecting consecutive points in the arrays. The style and colour of the line and point markers can be changed by inputting an optional third argument into the `plot` command. The commands `title`, `xlabel` and `ylabel` can be used to add a title and axis labels to the graph. After all the relevant graphing commands, the command `show()` must be used to display the graph.

(3) 1. One possible program is:

```
# Draw some breast cancer graphs
from pylab import *

t = arange(30, 86)

L=7.7*10**3*(85-t)*(2*t-59)
LDash=7.7*10**3*(229-4*t)
plot(t, L, label="L", linewidth=2)
plot(t, LDash, label="L'", linewidth=2)
title("Breast cancer graphs")
xlabel("Age (years)")
ylabel("Value")
grid(True)

# Draw the legend
legend(loc="center")
show()
```

2. Here are some properties:

- When the derivative is positive, the function increases (this happens for ages less than about 57).
- When the derivative is negative, the function decreases (this happens for ages more than about 57).
- When the derivative is 0, the function is level (this happens at about age 57).
- For a positive, decreasing derivative, the function increases at a decreasing rate.
- For a negative, decreasing derivative, the function decreases at an increasing rate.

(4) This is a discussion question.

(5) This is a discussion question.

(6) This is a discussion question.

- (7) The last page shows an example of a hand-written answer that would receive full marks on an exam.

Performance on this question was mixed. The concepts in it were completely new (that is, we had not covered them in class), but actually are not very hard. Also, it is possible to answer subsequent parts of the question without having answers to the earlier parts, and some parts are essentially 'common sense'. Your exam will include questions with many parts; if you cannot do some, that should not stop you attempting the later parts.

One thing to remember is: **don't write too much in your answers**. You do not need to answer in sentences, and you can use simple abbreviations and symbols, as long as it is clear what you mean. We will do our best to interpret your answers and understand what you are trying to communicate. Writing too much is a waste of time!

- (8) 1. In this answer, we will work in thousands of women.

For each age group 30–34, 35–40 and 40–44, there are between 150 and 162 thousand women, with an average of say 155. Noting that there are 5 years in each age group, there are about $155/5 = 31$ thousand women of each age. (Remember, this is just a model!)

For ages 45 and over, there appears to be a (rough) linear decrease in the number of women in each 5 year age group. For ages 45–50 the average number of thousands of women of each age is $160/5 = 32$. For ages 80–85, the number is $43/5 = 8.6$, which is close to 9. Thus, during the 35 years from age 45 to age 80, the average number of thousands of women of each age reduces by $32 - 9 = 23$, in 35 years, so the average annual decrease is $23/35 \approx 0.65$, which matches the gradient in the model.

Now it is simply a matter of finding the value of the constant in the equation of a straight line with gradient -0.65 , which equals 32 when $t = 45$. The equation is approximately $60 - 0.65t$, matching the model.

2. Here is one possible solution.

```
# Calculate YPLL from breast cancer in Qld.
from pylab import *

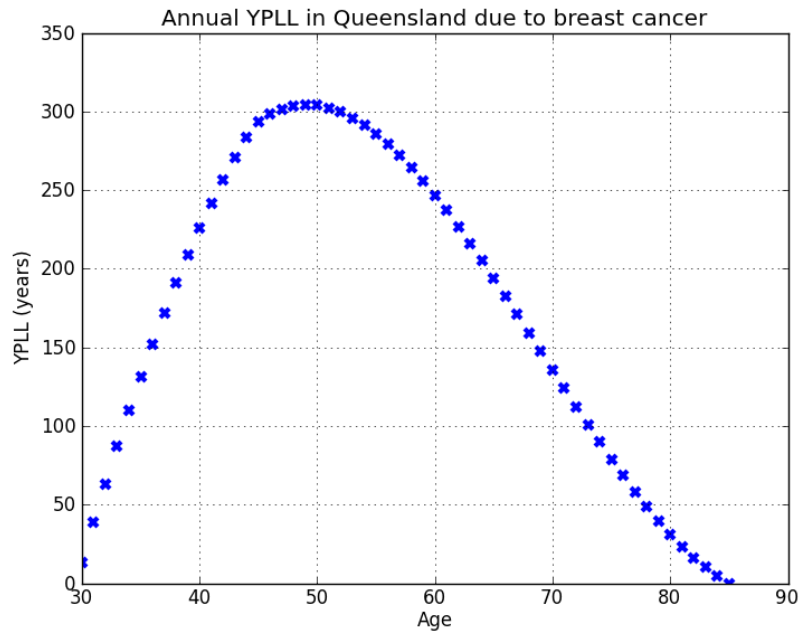
# YPLL for women aged under 45.
# Number of women of each age 30-45 is approx 31,000.
# Multiply YPLL per thousand women by 31.
t1=arange(30,44.5)
y1=7.7*10**-3*(85-t1)*(2*t1-59)*31
plot(t1,y1,'bx',mew=3)

# YPLL for women aged 45 and over.
# Number of women of each age x in thousands is approx 60 - 0.65 * x
# Multiply YPLL per thousand women by this.

t2=arange(45,85.5)
y2=7.7*10**-3*(85-t2)*(2*t2-59)*(60-0.65*t2)
plot(t2,y2,'bx',mew=3)
title("Annual YPLL in Queensland due to breast cancer")
xlabel("Age")
ylabel("YPLL (years)")

grid(True)
show()
```

The graph this program produces is:



3. The peak of this graph occurs around age 50, with total YPLL for each age 47 to 53 being very similar. The graph is not symmetrical about the maximum value, with instead a lengthy “tail” for larger ages. Recall that Queensland offers free breast screening for women aged 40 to 69. This age range captures a very large proportion of the total YPLL due to breast cancer.

- (9)
1. The derivative $W'(t)$ at time t is the instantaneous rate of change of the relative weighting of a lost year of life at age t . Its value is roughly the amount of increase or decrease in the weighting associated with that year.
 2. The physical interpretation of a positive derivative $W'(t)$ is that the person’s relative weighting is increasing. Thus, the person is “more valuable” next year than this year.
 3. The maximum of the graph of $W(t)$ will occur when its derivative is zero. So we want to find all times t when $W'(t) = 0$.

$$\begin{aligned} W'(t) &= 0.2e^{-0.04t}(1 - 0.04t) \\ 0 &= 0.2e^{-0.04t}(1 - 0.04t) \\ &= e^{-0.04t}(1 - 0.04t) \end{aligned}$$

Notice that $e^{-0.04t}$ is never zero, so we may divide both sides by this. Hence

$$\begin{aligned} 0 &= 1 - 0.04t \\ 0.04t &= 1 \\ t &= \frac{1}{0.04} \\ &= 25. \end{aligned}$$

At this stage we don’t know if this is a maximum or a minimum (or any other kind of stationary point). If $W(25)$ really is a maximum, then $W(25) \geq W(24)$ and $W(25) \geq W(26)$. Using a calculator, $W(25) = 1.8394$, $W(24) = 1.83789$ and $W(26) = 1.83796$, so $W(25)$ is in fact a maximum, and the maximum value is 1.8394.

4. From the table, A is $Z(t_0)$, and $t_0 = 10$, so

$$A = Z(10) = W(10) - 1 = 2e^{-0.4} - 1 = 0.3406.$$

From the table, $B = t_{0+1} = t_1$. From the lecture notes about Newton’s method, $t_{i+1} = t_i - \frac{f(x)}{f'(x)}$. In this case, our function is $Z(t)$ and the i we are concerned about to calculate B is 0. Thus

$$t_{0+1} = t_1 = t_0 - \frac{Z(t_0)}{Z'(t_0)}.$$

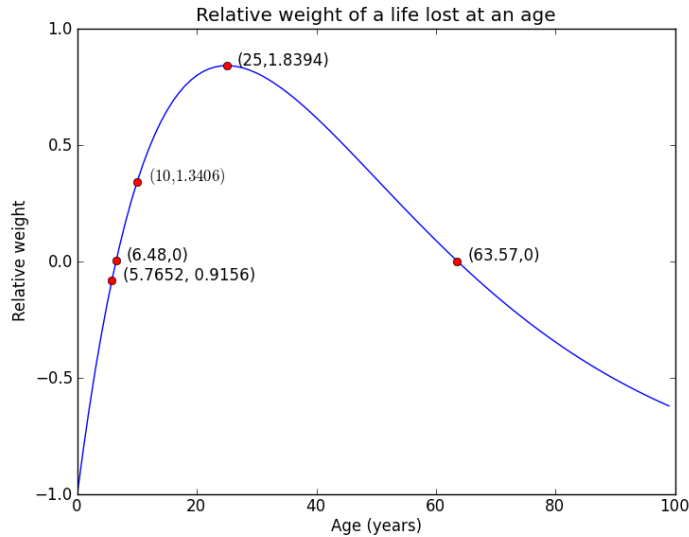


Figure 1: The relative weight of a life lost at each age from birth to 100 years old.

We don't need to calculate $Z(t_0)$ and $Z'(t_0)$ explicitly ourselves, as these values are in the table already ($A=0.3406$ and 0.0804 respectively). We may then calculate $t_1 = 10 - \frac{0.3406}{0.0804} = 5.7652$. From the table, $C = t_1$, but we know that $B = t_1$ also, so $C = 5.7652$ also. From the table, $D = t_{1+1} = t_2$. Again, $t_{1+1} = t_2 = t_1 - \frac{Z(t_1)}{Z'(t_1)}$, and again, $Z(t_0)$ and $Z'(t_0)$ are in the table already (-0.0844 and 1.222 respectively). We may then calculate

$$t_2 = 5.7652 - \frac{-0.0844}{1.222} = 6.4562.$$

5. These answers are the ages when the relative weighting of a year lost at that age is approximately 1. We already know there is exactly one maximum in the graph of $W(t)$, so it is reasonable for there to be two ages for which the relative weighting is 1, before and after the weighting reaches the maximum.
 6. See Figure 1.
 7. The graph starts at 0 because the 'social value' of a newly born child is zero. Throughout childhood, the child cannot look after itself, so society must invest much in the education and care for this child. The later into childhood the life is lost, the more investment society has put into the child, and the greater the relative weight of that loss. Once the life reaches 25, that person is typically a fully productive member of society. They help raise the next generation, and are in effect "paying back" the investment society has made in them. Thus, their relative weight goes down after they become a productive member of society. The graph does not reach 0 again during a human lifetime, suggesting that a person's life will never become worthless to society after reaching a certain age.
- (10) 1. Let t be the time at which $A(t)$ and $N(t)$ are the same. At this time, $A(t) = N(t)$, so we have that $15.6e^{0.024t} = 22.6e^{0.012t}$. Simplifying this equation gives:

$$\begin{aligned} \frac{15.6}{22.6} &= \frac{e^{0.012t}}{e^{0.024t}} \\ 0.69 &= e^{0.012t-0.024t} \\ 0.69 &= e^{-0.012t} \\ \ln(0.69) &= -0.012t \\ t &= 30.9 \end{aligned}$$

So the time at which $A(t)$ and $N(t)$ are the same is almost 31 years after 2010, which is almost the year 2041.

2. We have that

$$f(t) = N(t) - A(t) - 10 = 15.6e^{0.024t} - 22.6e^{0.012t} - 10.$$

From the hint, we can see that

$$f'(t) = 15.6 \times 0.024e^{0.024t} - 22.6 \times 0.012e^{0.012t} = 0.3744e^{0.024t} - 0.2712e^{0.012t}.$$

Each step of Newton's method is $t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)}$. Since the initial estimate is $t_0 = 0$, we can calculate

$$f(0) = 15.6e^0 - 22.6e^0 - 10 = -17$$

and

$$f'(0) = 0.3744e^0 - 0.2712e^0 = 0.1032.$$

So we have

$$\begin{aligned} t_1 &= 0 - \frac{f(0)}{f'(0)} \\ &= 0 - \frac{-17}{0.1032} \\ &= 165 \end{aligned}$$

So we predict after one iteration of Newton's method the time at which the population of Niger will be 10 million more than the population of Australia to be about 165 years after 2010, in the year 2175.

- (11) There is no trick to this question; just apply Newton's method. On the exam you will either be given the derivative, or told how to find it.

First, note that $f'(t) = e^t - 3$. Each step of Newton's method is:

$$t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)}.$$

Hence

$$\begin{aligned} t_1 &= 0 - \frac{f(0)}{f'(0)} \\ &= 0 - \frac{e^0 - 3 \times 0}{e^0 - 3} \\ &= 0 - \frac{1}{1 - 3} \\ &= 1/2. \end{aligned}$$

- (12) 1. The function for W_2 is a species area curve (the power is $1/2$), so grows indefinitely. However, the other function has a maximum "carrying capacity" of 10 (as t becomes large, the value approaches 10).
 2. Let $f(t) = W_2 - W_1 - 1$, so $f'(t) = W_2' - W_1'$.
 Hence $f'(t) = \frac{1}{\sqrt{t}} - e^{-0.1t}$.
 When $t = 1$ we have $f(t) = 2 \times 1 - 10(1 - e^{-0.1}) - 1 = -9 + 10e^{-0.1}$, and $f'(t) = 1 - e^{-0.1}$.
 Each step of Newton's method is:

$$t_{i+1} = t_i - \frac{f(t_i)}{f'(t_i)}.$$

$$\text{Hence } t_1 = 1 - \frac{-9 + 10e^{-0.1}}{1 - e^{-0.1}} = 0.492.$$

3. Newton's method uses an initial estimate to 'start' the iterations. Different initial estimates will result in different subsequent values. In this case, two different final estimates were obtained; this suggests that there are **two** values of t for which $W_2 - W_1 - 1 = 0$, the first being $t \approx 0.71059$ and the second $t \approx 2.13377$.
- (13) 1. The derivative $D'(t)$ shows the rate at which the proportion of people used illicit drugs in the last year is changing with age.

2. We want to find all t such that $0 = D'(t) = 4.8(t - 12)(22 - t)e^{-0.2t}$. By the null factor law, $D'(t) = 0$ only if at least one of the terms $t - 12$, $22 - t$ or $e^{0.2t}$ are 0. An exponential term can never be equal to 0, so the only two possibilities are $t = 12$ or $t = 22$. Since the model is only valid for $t < 12$, the only point in the range of the model is $t = 22$. We can now calculate the value of $D(t)$ at this age.

$$D(22) = 24(22 - 12)^2 e^{-0.2 \cdot 22} + 5 = 2400e^{-4.4} + 5 = 34.47.$$

This means that according to our model, 34.5% of people used drugs in the year they were 22, and this was the age when this proportion was the highest.

3. The output would be

```
22 14.466 0
```

4. A divide by 0 would occur, as v^2-0 .

5. Here is one possible solution.

```
# This program plots a model of how drug use varies with age
from pylab import *

# Create equally spaced points for age axis
t = arange(12, 83)

# Calculates D(t), the percentage of Australians of each age, t,
# from 13 to 82 who had used an illicit drug within the previous year
D = 24*(t-12)**2 * exp(-0.2*t) +5

# Creates graph, with necessary details
plot(t, D)
title("Australians who had used an illicit drug within the previous year")
xlabel("Age (years)")
ylabel("Percentage of Australians (%)")
show()
```

6. This graph has a steep gradient from ages 12 to 22, peaks at age 22, then has a slightly less steep decline after age 22. The graph flattens out into an asymptote as age increases. The asymptote is at the value 5%. This peak of the graph is expected, as drug use is most common in young adults, and the asymptote is expected, as drug use becomes less common in older populations, but is never completely absent.

- (14) 1. Here is a solution.

```
#A program to find sqrt 12
from pylab import *

# Initialise variables
ctr = 0
x = eval(input("What initial estimate for x? "))
maxIt = eval(input("How many iterations do you want? "))

# Loop through steps of Newton's method.
print(ctr, x)
while ctr < maxIt:
    ctr = ctr + 1

# Evaluate the function and its derivative at the current value of x
    f = x**2 - 12
    fDash = 2 * x

# Update x
    x = x - f / fDash
    print(ctr, x)
```

2. Here is the output. Estimated solutions are changing only very slightly after the first two or three iterations.

```
What initial estimate for x? 3
How many iterations do you want? 6
0 3
1 3.5
2 3.46428571429
3 3.46410162003
4 3.46410161514
5 3.46410161514
6 3.46410161514
```

(15) The only changed lines of the previous program are:

```
f = 1.4*pow(x,0.15)*exp(-0.02*x)-0.3
fDash = 1.4*exp(-0.02*x)*(0.15*pow(x,-0.85)-0.02*pow(x,0.15))
```

The output is:

```
What initial estimate for x? 50
How many iterations do you want? 6
0 50
1 89.7692171164
2 108.467471488
3 112.301955435
4 112.440256041
5 112.44042874
```

On the next page there is a hand-written solution to Question (7).

(a)(i) D is cumulative; must ↑

(ii) Age is a risk factor for b.c., so D↑ more rapidly as age ↑

$$(b) 7.7 \times 10^{-6} \times (60-30)^2 = 900 \times 7.7 \times 10^{-6}$$

(c) cumul. prob at age $(t+1)$ - cumul prob at age t

$$= 7.7 \times 10^{-6} \times ((t+1-30)^2 - (t-30)^2)$$

$$= 7.7 \times 10^{-6} \times ((t^2 - 58t + 29^2) - (t^2 - 60t + 30^2))$$

$$= 7.7 \times 10^{-6} \times (2t - 900 + 841) = 7.7 \times 10^{-6} \times (2t - 59)$$

$$(d) 1000 \times 7.7 \times 10^{-6} \times (2 \times 60 - 59)$$

$$= 61000 \times 7.7 \times 10^{-6} = 0.4697$$

(e) It allows a "cost" to be assigned to different diseases/accidents in different age groups, so society can do cost/benefit analyses of treatments/approaches.

$$(f) (85-60) \times 0.4697 = 9.394$$

(g) $L(t)$ is the number of deaths per 1000 women of a given age, multiplied by the YPLL per woman who dies.

(h) Society wants to spend money on fixing the

"greatest problem", where YPLL is highest: ages ~40 to ~69.

(i) $L' = 0 \Rightarrow$ graph has a peak, at $t \approx 58$ years.

This is the age at which the total cost of bc per 1000 women of that age has the highest loss of years.

$$(j) L' = 0 \Rightarrow 229 - 4t = 0 \Rightarrow t = 57.25 \text{ years, } \hat{\approx} 58 \text{ years.}$$

(k) eg $(85-t)^2$. For small t , this is much larger.