SCIE1000 Tutorial sheet 8

This tutorial contributes toward your final grade; see the Course Profile (https://www.uq.edu.au/study/course.html?course_code=SCIE1000). The tutorial will be marked out of 6, with 3 marks for completing the "Before class" work, and 3 marks for completing the "In class" assessment and working on the remaining "In class" questions until you finish them or the tutorial ends.

Goals: This week you will work through some calculation and discussion questions, mostly relating to quantifying the societal impact of disease and other similar factors that reduce life expectancy and quality of life. As usual, broad concepts and techniques are more important than specific examples. Do not try to commit lots of facts to memory; instead, know **how** to do things, and **when** certain models and approaches are appropriate. The aim of the computing component of this tutorial is to understand the concept of *plotting graphs*, and how you can customise the graphs produced by your Python programs.

To be completed before class

Complete the following questions before class, write (or type if you wish) your answers on a sheet of paper, put your name and student number on the top of the paper, and hand it to your tutor as you enter the room. **If you do not hand in the answers at the start of the class, as you enter the room, then you will lose the marks for this component.** Note that in some cases there are no "right" or "wrong" answers.

Question (1)

1. There are many ways of quantifying the societal impact of diseases and other causes of death or disability. These include measuring the *Number of Deaths*, *Mortality Rate*, *Years of Potential Life Lost* (YPLL or YLL), *Years Lost to Disability* (YLD) and *Disability Adjusted Life Years* (DALY). Find (and write down) good definitions of each of these quantities, including when it might be useful to measure that particular one.

2. The *World Health Organisation* (http://www.who.int/en/) (WHO) is conducting an ongoing, very large study into the *Global Burden of Disease* (http://www.who.int/topics/global_burden_of_disease/en/). Two recent publications associated with this study are *The global burden of disease: 2004 update (2008)* and *Global Health Risks (2009)*.

   Read **both** of the following quickly, and take some useful notes. (There may be questions relating to this on the final exam.) You do not need to note down details, but instead what are some of the key messages? Did you find anything surprising or interesting? What new things did you learn, if any? Do you think that DALY is a useful way of quantifying the societal cost of disease and other factors? Come along to class ready to discuss the answers to these questions.

   - Part 4 of *The global burden of disease: 2004 update (2008)* (http://www.who.int/healthinfo/global_burden_disease/GBD_report_2004update_part4.pdf?ua=1).
   - A Powerpoint summary of some data from *Global Health Risks (2009)* (http://www.who.int/entity/healthinfo/global_burden_disease/global_health_risks_report_figures.ppt?ua=1)

Question (2)

Read and understand Section A.7 in the Python Appendix of the lecture notes, covering "Graphs". (This material is in the book of lecture notes, near the end.) On your sheet of paper, write a short paragraph explaining, in general terms, how to plot graphs in your programs.

Question (3)

1. In lectures we modelled the probability of death from breast cancer prior to certain ages. Later in this tutorial sheet we will analyse this model further. Two functions we will encounter are

$$L(t) = 7.7 \times 10^{-3} \times (85 - t) \times (2t - 59)$$

   and its derivative,

$$L'(t) = 7.7 \times 10^{-3} \times (229 - 4t).$$

   Paste the following partial program into the Python cell below, and modify the program so that it plots a graph of L and L' on a single set of axes.

```
from pylab import *

t = arange(30,86)
L = .....
LDash = .....
plot(t, L, label="L", linewidth=2)
plot(t, LDash, label="L'",linewidth=2)

# Add title and labels
.....

grid(True)
# Draw the legend
legend(loc="center")
show()
```

2. The graph drawn by your program demonstrates a number of the features of a function and its derivative (for example, the derivative of a quadratic function is a linear function). Write down all of the other features of a function and its derivative that you can identify from the graph.

```
In [ ]: # Paste program here
```

To be completed in class

Complete the following questions in class. They involve a mix of individual work, and discussions with others. Make sure that you read the questions before class and think about how you might approach answering them. Don't rely on someone else doing all of the work. You need to work by yourself on the final exam, so it is important that you work hard now.

Feedback: Be proactive!

Australian government research shows that students often feel they don't receive adequate feedback on their work. In a class of 800 students, it is not possible for the course coordinator to give direct feedback to each student. Instead, tutorial classes are designed to be the place in which you can get feedback on your work from classmates and the tutors. You can ask for help, show them your answers, and discuss your understanding of any of the course material. As an adult learner, the onus is on **you** to seek feedback; tutors and classmates are happy to give it, if you want it.

Question (4)

Briefly discuss with a partner how to plot graphs. Ensure that you both agree on the key points.

Question (5)

Your tutors will record the marks for the sheet of paper you submitted with the "Before class" work, and they will then return your sheet to you early in the class so you can work from it.

1. Discuss your notes on the two WHO publications with your partner. If they have written something that you think may be important and that you missed, then update your notes.

2. Discuss the content of the publications as a group, and update your notes appropriately.

Question (6)

Discuss, with your partner, your answers to both parts of Question (3). Make sure you agree on all key points.

Question (7)

(Parts 1 to 9 of this question were on the final examination in 2011, and worth 22 marks. Expected working time for this question was about 22 minutes.)

In lectures, we used the following function $d(t)$ to model the probability that a woman will die of breast cancer **prior to reaching age t years**, where t is between 30 and 85:

$$d(t) = \frac{1}{43} \times \frac{1}{55^2} \times (t-30)^2.$$

Note that $d(t)$ simplifies (approximately) to:

$$D(t) = 7.7 \times 10^{-6} \times (t-30)^2.$$

1. Give one or more **physical** reasons why $D(t)$ is always:
   A. increasing as t gets larger; and
   B. increasing at an increasing rate as t gets larger.

2. Use $D(t)$ to estimate the probability of female death from breast cancer *before reaching* age 60 years.

3. Show that the expected probability that a woman will die from breast cancer **at** age t is equal to

   $$7.7 \times 10^{-6} \times (2t-59)\,,$$

   for $t \geq 30$. (Hint: calculate the cumulative probabilities of death prior to age t and age $(t+1)$.)

4. Find the expected number of **annual** breast cancer deaths in a group of 1000 women all aged 60. (Hint: refer to Part 3.)

5. When a person dies at an age younger than the population-wide life expectancy, the *Years of Potential Life Lost* (YPLL) equals the difference between the life expectancy and their actual life span. Why is YPLL a useful concept, and what role could it play in public health policy?

6. The life expectancy of Australian women is 85 years, so the YPLL for a woman who dies at age t years, $t \leq 85$, is $y(t) = (85-t)$. Estimate the **expected total YPLL due to breast cancer deaths in a year** for a group of 1000 women all aged 60. (Hint: refer to Part 4.)

7. Briefly justify why the expected total YPLL due to breast cancer deaths in a year for 1000 women of age t years is

   $$L(t) = 7.7 \times 10^{-3} \times (2t-59) \times (85-t).$$

8. In Queensland, all women aged from 40 to 69 are entitled to free breast cancer screening. Justify this, with particular reference to $L(t)$ (defined in Part 7). (Hint: first run the program in the following Python cell to plot the graph of $L(t)$.)

9. Let T be the time at which $L'(T) = 0$, where L' is the derivative of L. Find the (approximate) value of T using the graph of $L(t)$. Briefly interpret the physical meaning of your answer, and explain how it is relevant to society.

10. The derivative of $L(t)$ is

$$L'(t) = 7.7 \times 10^{-3} \times (229 - 4t).$$

Find the value of t for which $L'(t) = 0$ and compare this value with your answer to Part 9.

11. The equation for YPLL, $y(t) = (85 - t)$, values every year of potential lost life equally. Instead, governments often apply a *social weighting*, in which the **value** of each year of potential life lost depends on age. Commonly, a year lived as a young adult is valued more highly than a year lived as an older adult.

    Propose a new equation for $y(t)$ that gives a higher value to years lived as a young adult than to years lived as an older adult. Explain how your equation achieves this.

In [ ]:

```
# Run this program to plot a graph of L(t) from Question 7.
#
from pylab import *

t = arange(30,86)

L=7.7*10**-3*(85-t)*(2*t-59)
plot(t, L, label="L", linewidth=2)
title("YPLL due to breast cancer")
xlabel("Age (years)")
ylabel("YPLL (years per 1000 women)")
grid(True)
show()
```

Question (8)

This question is a required, in-class assessment piece. To receive the marks for this component, you and your partner must show your answers to a tutor during your tutorial.

From the previous question, the expected annual total YPLL due to breast cancer **for 1000 women** of age t years is

$$L(t) = 7.7 \times 10^{-3} \times (85 - t) \times (2t - 59).$$

Thus, the **total YPLL due to breast cancer per year for a geographical region** obviously depends on how many thousand women of each age live in that region. Run the Python program in the following cell to draw a frequency histogram showing the number of women living in Queensland in each 5 year age group from age 30 to 85, in thousands, using the most recently available *Australian census data* (http://www.abs.gov.au/ausstats/abs@.nsf/Products/3235.0~2011~Main+Features~Queensland?OpenDocument), conducted in 2011.

1. Use the graph to justify the following model briefly.

   The number of thousands of Queensland women of **each** age t from 30 to 44 is approximately equal to 31. The number of thousands of Queensland women of each age t from 45 to 85 is approximately equal to $60 - 0.65t$.

2. In the Python cell below the graph, write a program that calculates and draws a graph of the annual YPLL due to breast cancer in Queensland, for women of each age from 30 to 85. Your graph should have a meaningful title and labels on the axes. (Hint: your program should not be very long. You could consider modifying your program from Question 3. You program will still need to use $L(t)$, but not $L'(t)$. Your program will need to treat two separate ranges of ages.)

3. Briefly comment on your graph.

In [ ]:

```
# Plot a histogram of the QLD female population in 5 year age groups
from pylab import *

t = arange(30,85,5)
v = array([152.286,165.129,157.999,161.070,148.036,133.048,121.518,90.820,68.436,52.966,43.786])
bar(t,v,width=4.8)
title("Female population of Queensland")
xlabel("Age group")
ylabel("Number (thousands)")
grid(True)
show()
```

In [ ]:

```
# Write your program here
```

Question (9)

(This question was on the final examination in 2012, and worth 21 marks. Expected working time for this question was about 21 minutes.)

When a person dies from accident or disease, the *Years of Potential Life Lost* is the number of years the person died before reaching their life expectancy. Rather than assigning an equal "social cost" to each lost year of life, the social cost of each lost year is sometimes given a *relative weighting* depending on the age of the individual. The following function $W(t)$ gives a relative weighting of a lost year of life at age t, for t between 0 and 80 years, and $W'(t)$ is the derivative:

$$W(t) = 0.2te^{-0.04t} \qquad W'(t) = 0.2e^{-0.04t}(1 - 0.04t).$$

Part 4 of this question refers to the following table:

Step i	t_i	$Z(t_i)$	$Z'(t_i)$	t_{i+1}
0	10	A	0.0804	B
1	C	-0.0844	0.1222	D

1. What does the derivative $W'(t)$ represent?

2. What is the physical meaning of the ages at which the derivative $W'(t)$ is positive, and what does this mean in terms of social cost?

3. Find the age with the largest relative weighting, **and** find that weighting.

4. Consider the problem of finding a year for which the relative weighting equals 1, so $W(t) - 1 = 0$. Let $Z(t) = W(t) - 1$, so Newton's method can be used to solve $Z(t) = 0$. The above table shows partial calculations obtained when Newton's method is used with an initial estimate of $t_0 = 10$. Find the unknown values A, B, C and D in the table. Show your working.

5. Newton's method was run twice. With an initial estimate of $t_0 = 10$, after 4 steps, $t_4 \approx 6.48$ and is not changing significantly. With an initial estimate of $t_0 = 40$, after 4 steps, $t_4 \approx 63.57$ and again is not changing significantly. Explain briefly what these answers mean, and why there are two different values.

6. Recall that $W(t) = 0.2te^{-0.04t}$. Give physical reasons why it is reasonable for the weighting function for the social cost of *Years of Potential Life Lost* to have the shape of a surge function.

Extra questions

Here are some extra practise questions, for you to do in class (if you have time), or outside class. You do not need to do them all, but may like to choose some to help with your preparation for the final exam.

Question (10)

(This question was on the final examination in 2013, and worth 10 marks. Expected working time for this question was about 10 minutes.)

1. An exponential model of the Nigerien population $N(t)$ in millions at any time t in years since 2010 is

   $$N(t) = 15.6e^{0.024t}.$$

   The corresponding exponential model of the Australian population is

   $$A(t) = 22.6e^{0.012t}.$$

   Find the time at which $A(t)$ and $N(t)$ are the same.

2. Apply one step of Newton's method to predict the time at which the population of Niger will be 10 million more than the population of Australia. (Hint: Solve $N(t) - A(t) - 10 = 0$. Use an initial estimate of $t_0 = 0$. Note that the derivative of Ce^{kt} is kCe^{kt}.)

Question (11)

(This question was on the final examination in 2009, and worth 4 marks. Expected working time for this question was about 4 minutes.)

Consider the problem of finding a value of t for which $e^t - 3t = 0$. Apply one step of Newton's method to calculate t_1, using $t_0 = 0$. (Hint: if $f(t) = e^t - 3t$ then $f'(t) = e^t - 3$.)

Question (12)

(This question was on the final and deferred examinations in 2010, and worth 10 marks. Expected working time for this question was about 10 minutes.)

Researchers propose two models for estimating the mass of accumulated flammable material in a forest, where mass is measured in appropriate units and t is the number of years since the most recent bush fire. The models are:

$$W_1 = 10(1 - e^{-0.1t}) \qquad\qquad W_2 = 2\sqrt{t}.$$

1. The models produce very similar predictions for $t \le 20$. Explain mathematically why the predictions differ substantially for larger values of t.
2. Use **one** step of Newton's method to estimate a value of t at which $W_2 - W_1 = 1$. (Hint: let $f(t) = W_2 - W_1 - 1$, so you need to solve $f(t) = 0$. Use an initial estimate of $t = 1$ year, and note that $W_1' = e^{-0.1t}$ and $W_2' = \dfrac{1}{\sqrt{t}}$.)
3. Newton's method was twice used to solve $f(t) = 0$, with two different values of the initial estimate t_0, producing the results in the following table. Explain briefly why two different values were obtained for t_5, and what this means.

t_0	t_1	t_2	t_3	t_4	t_5
0.5	0.6587	0.7069	0.71057	0.71059	0.71059
4	2.2574	2.1379	2.1337	2.13377	2.13377

Question (13)

(Parts 1 to 4 of this question were on the midsemester examination in 2012, and worth 11 marks. Expected working time for this question was about 11 minutes.)

A recent publication gives the percentage of Australians of each age from 13 to 82 who had used an illicit drug within the previous year. If t is an age in years (so $t > 12$ and $t < 83$) then the following function $D(t)$ gives a good model of the drug usage data; the derivative $D'(t)$ is also given.

$$D(t) = 24(t-12)^2 e^{-0.2t} + 5$$

$$D'(t) = 4.8(t-12)(22-t)e^{-0.2t}$$

1. What does the derivative $D'(t)$ represent, in terms of illicit drug usage?

2. Find the age(s) at which $D'(t)$ equals zero and find the corresponding value(s) of D. Briefly interpret your answer. (Hint: note that D and D' have only been defined for $t > 12$.)

3. When asked to write a program to use Newton's method to find the age at which $D(t) = 20$, a student comes up with the following partial program. Write down all of the output from the partial program.

```
def Ddash(t):
    d = 4.8*(t-12)*(22-t)*exp(-0.2*t)
    return d

def D(t):
    d = 24*(t-12)**2 * exp(-0.2*t) - 15
    return d

initT = 22
v1 = D(initT)
v2 = Ddash(initT)
print(initT, v1, v2)
```

4. The student then **adds** the following additional new line to the **end** of the program. What additional output does the program produce? Explain your answer briefly.

```
print(initT - v1/v2)
```

5. In the following Python cell, write a program to plot the graph of $D(t)$, with an appropriate title and labels on the axes.

6. Explain the general shape of the graph of $D(t)$.

In [ ]:
```
# Write your program here
```

Question (14)

1. Write a Python program to implement Newton's method to estimate $\sqrt{12}$, as was done in the lecture notes. Start by studying the formal description of the algorithm in the lecture notes: you basically need to "translate" the English description of the algorithm into Python. Think about the programming techniques that you have used in previous weeks. Do you need a loop? What variables do you need? You may need to ask the user to input the maximum number of steps to use. Make sure you write your program so that it can be easily modified to solve different functions.

2. Run your program. Your result should be identical to the table in the notes (for two iterations). Run your program for a larger number of iterations and observe how the solution value changes.

In [ ]:
```
# Write your program here
```

Question (15)

In lectures we used Newton's method to find the time at which the blood concentration of an injected hormonal contraceptive fell below the level required for effective contraceptive function (0.3 ng/mL). To do this we solved $f(t) = 0$, where

$$f(t) = 1.4t^{0.15}e^{-0.02t} - 0.3$$

$$f'(t) = 1.4e^{-0.02t}\left(0.15t^{-0.85} - 0.02t^{0.15}\right)$$

Finally, we used $t_0 = 50$ as the initial estimate for the solution.

Modify your previous Newton's method program to find an approximate solution to this problem. (You should find that after 4 or 5 steps, the solution is $t \approx 112.44$ days.)

In [ ]:
```
# Write your program here
```