SCIE1000, Solutions to Tutorial Week 9.

(1) 1. Here are some quotes/points made in the article; comments on how it made the reader feel and what it made the reader think will obviously vary between individuals.

   - The more the defense lawyer can shake jurors loose from the predisposition of guilt ... the more favorable the trial prospects of the defense. Attacking Widmark calculations is an effective way to accomplish this mission because the Widmark formula uses averages to arrive at conclusions.
   - When jurors hear that there is going to be a calculation using a long established formula, most jurors presume that the calculation is going to have some type of mathematical certainty. However, when the jurors hear that in reality the Widmark calculations are actually nothing more than an educated guess, hopefully they will begin to question some of their initial assumptions.
   - One way to remind jurors about the problems with averages that are used in Widmark calculations is to give them analogies that emphasize the problem with using averages ... For instance, you might tell jurors if one leg on a mans pants is too short, and one leg is too long, the prosecutions tailor would have you believe that on average the pants fit just fine.
   - Many experts do not really have an expert understanding of Widmark's work. What they count on is that you do not have much of an understanding of the work either. Many experts will ... use a shorthand version of the formula in court. This shorthand version of the formula does not accommodate any of the variables that are in the actual formula... The formula is the number of drinks, times 3.75, divided by your client's weight, minus 0.02% per hour.
   - Most experts have not read Widmark's work.
   - There are two variables in the Widmark formula. ... The important thing for lawyers to know about variables is that the numbers that can be placed in them are, by definition, variable. That is, they are not constant for all people. However, the shorthand formulas that the prosecution's experts typically use assume constant variables for all people. These formulas assume that the defendant has an average r factor... In essence, the prosecution criminalist makes an educated guess about the defendant's drinking.

2. The answers to this question are a matter of personal view.
3. The answers to this question are a matter of personal view.

(2) 1. It is **not** due to lower population, or fewer cars, or less driving; those factors have all increased substantially. Reasons could be:

   - Improved safety of motor vehicles
   - Improvements to road infrastructure
   - Better education regarding risk factors (alcohol, speed, drugs)
   - Better training of drivers
   - Tougher road rules (eg reduced speed limits) and stronger penalties

2. From the graph there were about 3200 fatalities in 1982 and 1200 in 2013. The population has increased by a factor of $23.8/15.2 == 1.565$. Hence the number of fatalities would have been $3200 \times 1.565 = 5010$. In reality there were about 1200 deaths, so about $5010 - 1200 = 3810$ lives were saved.
3. From the graph, the y-intercept is 3000. The line goes down from 3000 to 1200, in 33 years, so the gradient is $1800/33$, and must be negative.
4. It's really not clear which model will be better in the future, because it will depend on many factors (such as population growth, technological advances, and possibly reduced availability of petrol). However, it would seem unlikely that the number would keep dropping at a linear rate, or at least at the same linear rate. Thus, the exponential model will probably be more accurate, and the predicted number of fatalities in 2030, which corresponds to $t = 48$, is about 1070.

(3) Arrays store values, as do other variables. However, arrays can store **multiple** values at the same time, rather than just a single value. Thus, arrays are a bit like tables of data. Hence you need to know the *location* of an item in the array. This location is given by the *index* of that entry in the array.

(4) This is a discussion question.

(5) This is a discussion question.

(6) 1. Here is the output:

```
[0 3 6 9 12]
```

2. Here is a solution.

```
from pylab import *

t = eval(input("How many hours: "))

BACArray = zeros(t+1)

# Fill the BAC array
n=0
while n <= t:
    if n<5:
        BACArray[n] = n
    else:
        BACArray[n] = 10
    n = n + 1
```

(7) This question is not hard, but involves logically stepping through the calculations to work out what is happening. The questions provide a sensible structure for thinking about the situation.

1. We know that the person died EXACTLY 3 days earlier, or 72 hours before the sample was taken. At the time of the sample, the person's BAC was 0.047%. We also know that ethanol stored under these conditions has a (post-mortem) decay constant of $k = 0.00111$ hrs^{-1}. Using this information we get:

$$\begin{aligned} A(72) &= A_0 e^{-0.00111\times 72} \\ 0.047\% e^{0.00111\times 72} &= A_0 \\ A_0 &\approx 0.0509\% \end{aligned}$$

So the person's blood alcohol content A_0 at the time of death was approximately 0.051%, which is over the legal driving limit.

2. Assuming he commenced driving **exactly** one hour before his death, we can use the Widmark formula to determine how much alcohol was in his system at the time he started driving. From lectures, we have seen that the rate at which the body metabolizes ethanol is 0.015% per hour. Therefore, if he was driving for one hour when he died, and his BAC at the time of death was 0.051%, his BAC when he commenced driving was approximately $0.051 + 0.015 = 0.066\%$.

3. Note that for this man, $r = 0.7$. Employing the Widmark formula, we can estimate the amount of ethanol the individual had in his system when he stopped drinking. So, given that he stopped drinking three hours prior to death we have:

$$\begin{aligned} \text{BAC} &= \frac{A}{rW} - 0.015t \\ 0.051\% &= \frac{A}{0.7 \times 80000 \text{ g}} \times 100\% - 0.015 \times 3 \\ 0.051 + 0.045 &= \frac{A}{0.7 \times 80000 \text{ g}} \times 100\% \\ A &= 0.096 \times \frac{(0.7 \times 80000 \text{ g})}{100\%} \approx 53.76 \text{ g of alcohol.} \end{aligned}$$

Since we know that one standard drink is approximately 10 grams of alcohol, we can estimate that the individual had consumed about 5.4 standard drinks. Therefore the witness statements are obviously incorrect or false, and one must call into question the reliability of their other statements.

item[(8)] This question (and hence the program) is a bit tricky. The key is to START at the time of the post mortem tissue sample, then extrapolate BACK. First, we deduce the BAC at time of death using an exponential decay function with decay rate of 0.00111 % per hour, as calculated on the tutorial sheet this week. Then, when the BAC at time of death is known, extrapolate BACK once again, using a constant metabolism rate of 0.015 % per hour.

Here is a solution.

```
# Program to calculate BACs both pre and post mortem.
from pylab import *

# Input data
print("In all cases, t=0 when drinking ceased.")
tDeath = eval(input("At what time did death occur: "))
tPM = eval(input("At what time was the post mortem tissue sample analysed: "))
PMBAC = eval(input("What was the post mortem BAC: "))

# Calculate the BAC at time of death.
tDead = tPM - tDeath
deathBAC = PMBAC/exp(-0.00111 *  tDead)

# Calculate the initial BAC (we assume alcohol is metabolised at 0.015 %/hr).
initBAC = deathBAC + 0.015 * tDeath

# Print BACs at t=0 and at time of death.

print("BAC at t=0 was",initBAC)
print("BAC at time of death was",deathBAC)

# Create array of times for graph and and empty array for BAC
timeArray = arange(0,tPM+1)
BACArray = zeros(tPM+1)

# Fill the BAC array
n=0
while n <= tPM:
    if timeArray[n] > tDeath:
        BACArray[n] = deathBAC * exp(-0.00111 * (n - tDeath))
    else:
        BACArray[n] = initBAC - n * 0.015
    n = n + 1

# Plot the graph
plot(timeArray, BACArray)
title("BAC of victim from when drinking ceased until time of post mortem")
xlabel("Time since drinking ceased (hours)")
ylabel("BAC")
show()
```

(9) Add the following lines to the program, just above where the graph is plotted.

```
# Estimate the number of standard drinks consumed.

Gender = eval(input("Type 1 for male, anything else for female: "))
mass = eval(input("Enter mass in kg: "))
if Gender == 1:
    r = 0.7
```

```
else:
    r = 0.6
A = initBAC * r * mass * 1000 / 100
print("The estimated number of standard drinks is", A/10)
```

item Here is the program output:

```
In all cases, t=0 when drinking ceased.
At what time did death occur: 3
At what time was the post mortem tissue sample analysed: 75
What was the post mortem BAC: 0.047
BAC at t=0 was 0.0959104191843
BAC at time of death was 0.0509104191843
Type 1 for male, anything else for female: 1
Enter mass in kg: 80
The estimated number of standard drinks is 5.37098347432
```

Hence the BAC at time of death was ≈ 0.0509 %, and the BAC when he ceased consuming alcohol was ≈ 0.0959 %. His BAC when he commenced driving is $0.0509+0.015 = 0.0659$ %. The number of standard drinks consumed was about 5.4. This output matches the results from the earlier hand calculations.

(10) This is standard high school mathematics.

1. Find the derivative B', then find the time t at which $B' = 0$.
2. Substitute in the values, making sure that the units are correct (for exampleb$M = 107000$ g, $A = 40$ g, and so on. With food in the stomach, $k = 2.3$ and $t_{max} = 0.933$ hr, or 56 mins. With an empty stomach, $k = 6$ and $t_{max} = 0.518$ hr, or 31 mins.
3. The value of B_{max} may be found by substituting t_{max} into the formula for BAC. With food in the stomach, $B_{max} \approx 0.035$ %. With an empty stomach, $B_{max} = 0.046$ %. We can see that B_{max} for the case of no food in the stomach is almost 33% higher than for the case in which there is food in the stomach.

(11) Here are the two sets of output.

First program:

```
40.0 77.0
50.0 308.0
60.0 693.0
70.0 1232.0
```

Second program:

```
h =  2  and a[i] =  2400.0
h =  2  and a[i] =  1600.0
h =  4  and a[i] =  3200.0
h =  2  and a[i] =  2000.0
AUC =  9200.0 units.
```

(12) 1. $PV = nRT \Rightarrow T = \dfrac{PV}{nR}$

So, $\dfrac{dT}{dt} = \dfrac{d}{dt}(\dfrac{PV}{nR}) = \dfrac{P'V + PV'}{nR}$ by the product rule (note that n and R are both constant).

Now, we know that $P' = 0.1$ atm/min, $P = 8$ atm, $V = 10$ L, $V' = -0.15$ L/min, $n = 10$ mol and $R = 0.0821$ atm.L/K/mol.

Hence, $\dfrac{dT}{dt} = \dfrac{0.1 \text{ atm/min} \cdot 10 \text{ L} + 8 \text{ atm} \cdot (-0.15) \text{ L/min}}{10 \text{ mol} \cdot 0.0821 \text{ atm.L/K/mol}} \approx -0.243605$ K/min.

2. The units are K/min, which certainly makes sense for the rate of change of temperature.

(13) Here are the three sets of output.

First program:

```
0 0.0
1 1.0
2 4.0
3 9.0
cs = [ 0.  1.  4.  9.]
```

Second program:

```
A = [14  4  4  8 40]
```

Third program:

```
[ 1.  1.  2.  3.  5.  8.  0.  0.  0.  0.]
```

(14) There is a slight trick here; the minimum occurs when the derivative equals 0, not when the function equals 0.

We wish to use one step of Newton's method to find an estimate for where the minimum of $f(t) = t^2 + e^{-t}$ occurs. To do this, we use Newton's method to solve $f'(t) = 0$ rather than $f(t) = 0$.

Using $f'(t) = 2t - e^{-t}$ and $f''(t) = 2 + e^{-t}$, and taking $t = 0$ as our initial estimate, we get:

$$\begin{aligned} t_1 &= t_0 - \frac{f'(t_0)}{f''(t_0)} \\ &= 0 - \frac{2 \times 0 - e^0}{2 + e^0} \\ &= \frac{1}{3} \end{aligned}$$

So a better estimate for the value of t at which the minimum of the function occurs is $t = \frac{1}{3}$. (For reference, the true value of the minimum is $t \approx 0.3517\ldots$ which means our one-step Newton's method estimate is accurate to within 5.3%.)

(15) Here is the output:

```
2
1
20
2
[ 3  6  9 12 15]
```