

## SCIE1000, Solutions to Tutorial Week 2.

- (1) This question is not meant to be hard. There are many ways that you could approach this question. You could draw a picture if that helps, or even ‘draw a picture in your mind’. Roughly what shape is a human? Remember that you need to estimate the BSA and volume of the human, so you should pick a shape for which you know how to calculate these things. Any rough, yet reasonable, model is acceptable. The most common answers are to model the human as a cylinder or a rectangular prism.

In this answer, we model the human as a cylinder (ignoring the top and bottom), with a height  $h$  and radius  $r$ . It seems reasonable to ignore the top and bottom in the BSA calculation, as they are small compared to the side of the cylinder. Furthermore, the body tapers at the head and feet, so using a cylinder of the same height as the person should overestimate the BSA, so we can ‘afford’ to lose the top and bottom.

Then using this model:

$$BSA = 2\pi rh \quad \text{and} \quad V = \pi r^2 h$$

However, we cannot easily measure the ‘radius’ of a human with a tape measure. What we can measure is the circumference. The circumference will be different at different places in the body, so to get a reasonable estimate, make sure not to measure the circumference at the widest or skinniest parts of the body.

We know that the circumference,  $c$ , of a circle is  $2\pi r$ , which means that  $r = \frac{c}{2\pi}$ . Thus, our model becomes

$$BSA = 2\pi \frac{c}{2\pi} h \quad \text{and} \quad V = \pi \left(\frac{c}{2\pi}\right)^2 h$$

which simplifies to

$$BSA = ch \quad \text{and} \quad V = \frac{hc^2}{4\pi}.$$

- (2) 1. The models are:

$$T = -794.4 + 1267.3B \quad T = 191.8 + 18.51W$$

where  $T$  is the TLV in  $\text{cm}^3$ ,  $B$  is the BSA in  $\text{m}^2$  and  $W$  is the weight in kg.

2. By examining Figure 1, it is fairly clear that the largest difference occurs for the individual with TLV about  $3500 \text{ cm}^3$ , and BSA of about  $2 \text{ m}^2$ . The modelled TLV is  $T = -794.4 + 1267.3 \times 2 = 1740 \text{ cm}^3$ . This is an error of about 100%.
3. Remember, a model is just an approximation to the ‘real world’, that allows useful conclusions to be drawn. Clearly, this individual has a reasonably small BSA, but a very large liver. This could be genetic, or maybe induced by lifestyle factors. It does not mean the model is useless (because it provides a good estimate of TLV for many other members of the study), but the model should not be assumed to be completely accurate if and when it is used in practice.
- (3) 1. Answers will vary, depending on your individual models. Basically, you simply replace BSA in the first TLV formula with your model for BSA, and in the second TLV formula you replace weight by your model for BV, multiplied by 1000 to convert from volume to weight (in fact, mass).
2. Clearly, heavier people will tend to have larger BSA and BV. Because the liver undertakes a number of important roles for the body, a larger body will also tend to be associated with a larger liver, and hence increased TLV. The converse is likely to be true for smaller people.
- (4) Variables are extremely important in programming. A basic calculator can only operate on one number at a time, the number on the screen, and rarely has the capacity to ‘remember’ numbers used previously. However, a computer program often needs to be able to remember many values at a time. To ‘save’ a particular value in a program, the value is assigned to a variable, which is usually a word or an abbreviation which describes this value. To use this value again in later calculations, the variable’s name is used in the calculation, but the computer treats it in the calculation as if it were the number that was assigned to it earlier.

- (5) Here is a possible solution.

```
# A mass/height program.
from pylab import *

massKG = 38.5
heightCM = 155
print("Mass =", massKG)
heightM = heightCM/100
print("Height in m = ", heightM)
BMI = massKG / heightM**2
print("BMI = ", BMI)
```

Here is the output.

```
Mass = 38.5
height in m = 1.55
BMI = 16.0249739854
```

(6) This is a discussion.

(7) 1. This is a discussion.

2. The tutor may be about 1.7 m tall. Imagine them as a cylinder, about 0.8 m in circumference. Using the model developed in Question (1), we would calculate

$$BSA = 0.8 \times 1.7 \approx 1.4\text{m}^2 \quad \text{and} \quad V = \frac{1.7 \times 0.8^2}{4\pi} \approx 0.087\text{m}^3.$$

As humans are mostly water, it is reasonable to approximate the density of a human as the density of water, which is 1kg/L. So, a volume of  $0.087\text{m}^3$  would mean a weight of about 87kg. This is a reasonable mass for a human, so our model appears to give reasonable results.

3. The ratio of the two expressions in our model gives the volume-to-BSA ratio:

$$\frac{V}{SA} = \frac{\pi r^2 h}{2\pi r h} = \frac{r}{2}.$$

The units of volume are  $\text{m}^3$  and the units of BSA are  $\text{m}^2$ . Hence the units of the ratio are m, which may be confusing, or  $\text{m}^3/\text{m}^2$ , which may make more sense.

4. To answer this question you will need reasonable estimates of the heights and radii of the cylinders used to model each person. Table 1 shows estimated values, followed by the table of answers using the example model.

Comparison of approx. V/SA ratio versus age					
Age	height	radius	Volume ( $\text{m}^3$ )	BSA ( $\text{m}^2$ )	ratio
1 yr	$\approx 0.60$ m	$\approx 0.05$ m	0.0047	0.1885	0.025
17 yr	$\approx 1.70$ m	$\approx 0.12$ m	0.0769	1.282	0.06
50 yr	$\approx 1.70$ m	$\approx 0.14$ m	0.1047	1.495	0.07

Table 1: Comparison of approximate volume-to-surface-area ratios over different people.

The ratios show that as a person gets bigger, the ratio of volume to surface area increases. For interest, noting that the human body is mostly water (so the mass of  $1\text{m}^3$  is approximately 1000 kg), the cylindrical model of volume predicts the baby has mass about 4.7 kg, the 17 year old has mass 77 kg and the 50 year old has mass 105 kg. These are not unreasonable estimates.

5. This is a discussion.

6. There are many possible answers, such as:

- The need to keep babies warm, as they have a large surface area compared to their volume so can lose a lot of body heat.
- The tendency for cold-climate animals to be 'more spherical', so they have a large volume compared to surface area, which means they lose less heat.

- A limit on the size of cells in an organism, as water and nutrients need to diffuse across the cell wall.
- Any object which is spherical or round, as that shape maximises volume to surface area.
- In two dimensions, any object that is circular, as that shape maximises area to perimeter (which is similar to volume and surface area).

(8) This question is pretty easy: there is no need to think much about what is needed, as values can be substituted straight into the given formulae and the model we developed above.

1.2. The table shows the estimates of the BSA for Nicole Richie, calculated using the two given formulae. For our own model developed above, we use a height of 155 cm and a radius of 0.09 m (this gives a volume for Nicole Richie of 0.039 m<sup>3</sup>, which corresponds roughly to a mass of about 39 kg, which is correct for her).

Comparison of BSA predictions	
Model	BSA
Our model	0.88 m <sup>2</sup>
Mosteller formula	1.29 m <sup>2</sup>
Dubois & Dubois formula	1.31 m <sup>2</sup>

Table 2: Comparison of BSAs for Nicole Richie, calculated using three approaches.

- Looking at the results in the table, we can see there is only a very minor variation in the results for the two “official” formulae. However, our model produces an estimate that is about 33% smaller than the other estimates, which is a fairly substantial difference. Hence our model appears to be not particularly accurate.
- We estimated Nicole Richie’s volume as 0.039 m<sup>3</sup>, so we are considering a sphere of this volume. We have volume, and we want to find radius, so we will rearrange the formula for the volume of a sphere to give

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Subbing in our volume estimate gives

$$r = \sqrt[3]{\frac{3 \times 0.039}{4\pi}} \approx 0.21\text{m.}$$

The surface area of a sphere with this radius is

$$SA = 4\pi r^2 = 4\pi(0.21)^2 \approx 0.56\text{m}^2.$$

(9) Here is a possible solution.

```
#Answers to various calculations for BSA
from pylab import *

#part i
i = 0.0167 * sqrt(38.5 * 155)
print("the answer to Part i) is ",i)
#part ii
ii = 0.007184 * 38.5**0.425 * 155**0.725
print("the answer to Part ii) is ",ii)
```

Here is the output.

```
the answer to Part i) is 1.29006824432
the answer to Part ii) is 1.31273666358
```

(10) Here is a possible solution.

```

# A program to estimate body surface area (BSA) in two ways.
from pylab import *

mass = 38.5
heightCM = 155

# Estimate BSA using Mosteller formula
BSAMos = 0.0167 * sqrt(mass * heightCM)

# Estimate BSA using Dubois & Dubois formula
BSADub = 0.007184 * mass**0.425 * heightCM**0.725

# Print output
print("(All measurements are in m^2.)")
print("Estimated BSA using Mosteller formula, is ",BSAMos)
print("Estimated BSA using Dubois & Dubois formula is ",BSADub)

print("The difference (Mosteller minus Dubois) = ", BSAMos - BSADub)

```

**Here is the output:**

```

(All measurements are in m^2.)
Estimated BSA using Mosteller formula, is 1.29006824432
Estimated BSA using Dubois & Dubois formula is 1.31273666358
The difference (Mosteller minus Dubois) = -0.0226684192567

```

The values from the program match the results on the tutorial sheet.

**(11) Replace the two lines**

```

mass = 38.5
heightCM = 155

```

by two new lines, such as:

```

mass = eval(input("What is the mass (in kg)? "))
heightCM = eval(input("What is the height (in cm)? "))

```

When 38.5 kg and 155 cm are entered as the mass and height, the same output is obtained as above.

**(12) Here is a possible solution.**

```

# A program to estimate body surface area (BSA) in two ways.
from pylab import *

print("This program estimates Body Surface Area (BSA) in two ways.")

mass = eval(input("What is the mass (in kg)? "))
heightCM = eval(input("What is the height (in cm)? "))

# Estimate BSA using Mosteller formula
BSAMos = 0.0167 * sqrt(mass * heightCM)

# Estimate BSA using Dubois & Dubois formula
BSADub = 0.007184 * mass**0.425 * heightCM**0.725

```

```
# Print output
print("(All measurements are in m^2.)")
print("Estimated BSA using Mosteller formula, is ",BSAMos)
print("Estimated BSA using Dubois & Dubois formula is ",BSADub)

print("The difference (Mosteller minus Dubois) = ", BSAMos - BSADub)
```

(13) This question was done very poorly on the exam. There were more parts to it; they are included on a later tutorial sheet. It seems likely that most students did not take the time to read what was being asked, or to think about it. Essentially, the question is fairly similar to the model we developed for volume above. The question starts with some unit calculations and conversions, then uses a cylinder to model a pig.

1. Noting that  $p$  is measured in pounds, and  $x$  and  $y$  are measured in inches, then working with units we have:

$$\text{pounds} = \frac{\text{inches}^3}{400}.$$

The only place that there can be missing units is on the constant 400. Hence its units must be  $\text{inches}^3 \text{pounds}^{-1}$ .

2. Writing an equation, with units in it, gives

$$p \text{ pounds} = M \text{ kg} \times 2.2 \text{ pounds kg}^{-1}$$

and the units resolve correctly.

3. Using a similar approach to Part 2, we have:

$$x \text{ inches} = G \text{ m} \times 39.4 \text{ inches m}^{-1}$$

$$y \text{ inches} = L \text{ m} \times 39.4 \text{ inches m}^{-1}$$

Again, the units resolve correctly.

4. We have

$$p = \frac{x^2 y}{400}, \quad \text{so} \quad 2.2M = \frac{(39.4G)^2 \times 39.4L}{400}, \quad \text{so} \quad M \approx 69.5G^2L.$$

5. Note that the question gives you all of the formulae that you need; you will not need to commit such things to memory for the exam.

First, note that we need to know the radius  $r$ , but we instead have the girth  $G$ . Recalling that the girth is the circumference, we have  $G = 2\pi r$ , so  $r = G/2\pi$ . Hence the formula for the volume of a cylinder of girth  $G$  and length (height)  $L$  is:

$$V = \pi \left( \frac{G}{2\pi} \right)^2 L = \frac{G^2 L}{4\pi}$$

where the units of  $V$  are  $\text{m}^3$ . Now, to convert from  $V$  to mass  $M$ , multiply  $V$  by  $1000 \text{ kg m}^{-3}$ , giving

$$M = \frac{1000G^2L}{4\pi} \approx 79.6G^2L.$$

Of course, on an exam you won't be able to type your answers. Also, in the above solutions, we have given lots of details so you can understand what is happening. For reference, the following (shorter and messier) answer would receive full marks on the exam. (Pay careful attention to the solution; we didn't hand-write it because we were too lazy to type the answer. Instead, it should give you a good idea of how little you need to write, how 'messy' you can be, and how you can use abbreviations.)

$$(a) \text{ pounds} = \frac{\text{in}^3}{600 \text{ units}} \quad \text{so} \quad 600 \text{ units} = \frac{\text{in}^3}{\text{pounds}}$$

$$(b) p \text{ pounds} = M \text{ kg} \times 2.2 \text{ pounds/kg}$$

$$(c) \begin{aligned} x \text{ in} &= G \text{ m} \times 39.4 \text{ in/m} \\ y \text{ in} &= L \text{ m} \times 39.4 \text{ in/m} \end{aligned}$$

$$(d) \begin{aligned} p &= \frac{\rho L^2 y}{400} \\ \rightarrow 2.2M &= \frac{(G \times 39.4)^2 L \times 39.4}{400} \\ &= \frac{G^2 L \frac{39.4^3}{2.2 \times 600}}{400} \\ &= 69.5 a^2 L \end{aligned}$$

$$(e) \begin{aligned} G &= 2\pi r \rightarrow r = G/2\pi \\ \pi r^2 h &\rightarrow \pi \times \left(\frac{G}{2\pi}\right)^2 \times L \\ V &= \frac{G^2 L}{4\pi} \text{ m}^3 \\ \Rightarrow M &= \frac{1000 G^2 L}{4\pi} = 79.6 a^2 L \\ \text{kg/m}^3 \times \text{m}^3 &= \text{kg} \end{aligned}$$

- (14) 1. Let Niger's annual CO<sub>2</sub> emissions from using petrol in 2010 be represented by A. We know how much petroleum Nigeriens used per day in 2010, so multiplying by the number of days in the year will give the total amount of petroleum used. All we then need to do is multiply this by the amount CO<sub>2</sub> emissions produced by one barrel of oil. These calculations can be performed in one line:

$$\begin{aligned}
 A &= \frac{5600 \text{ barrels}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}} \times \frac{159 \text{ litres}}{1 \text{ barrel}} \times \frac{2.2 \text{ kg}}{1 \text{ litre}} \\
 &= 715 \times 10^6 \text{ kg year}^{-1}
 \end{aligned}$$

So Niger's annual CO<sub>2</sub> emissions from using petrol in 2010 was about 715 million kg.

2. To calculate Niger's annual per-capita electricity usage, we just divide the total annual Nigerien electricity usage by the Nigerien population. This gives

$$\frac{702 \text{ million kWh}}{15.6 \text{ million people}} = 45 \text{ kWh person}^{-1},$$

which is value we were asked to find. The units of this answer are kWh/person.

(15) Here are the errors:

- The line that commences `First` is a comment, so must commence with `#`
- The mass in pounds is calculated incorrectly, and should be `MassPd = x*x*y / 400`
- The mass in kg is calculated incorrectly, and should be `MassKG = MassPd / 2.2`
- The final print statement is missing closing quotes after `The mass is`
- The final print statement needs to print `MassKG` not `MassPd`.

Here is a corrected version of the program:

```
# This program estimates the mass of a pig using a "traditional" method.
from pylab import *
print("---Welcome to PigCalc v1.0---")

# First, input the mass of the pig.
x = eval(input("Please enter the girth of the pig (in inches): "))
y = eval(input("Please enter the length of the pig (in inches): "))
MassPd = x * x * y / 400
MassKG = MassPd / 2.2
#Finally, print the results
print("The mass is",MassKG, " kg")
```

Here is the output for a pig with girth 54 inches and length 48 inches.

```
---Welcome to PigCalc v1.0---
Please enter the girth of the pig (in inches): 54
Please enter the length of the pig (in inches): 48
The mass is 159.054545455 kg
```

- (16) Rearranging the Hagen-Poiseuille equation gives  $\mu = \frac{\Delta P \pi r^4}{8QL}$ .

Rewriting the right-hand side in units and simplifying gives:

$$\text{Units of } \mu = \frac{\text{Pa m}^4}{\text{m}^3 \text{ s}^{-1} \text{ m}} = \frac{\text{Pa m}^4}{\text{m}^4 \text{ s}^{-1}} = \text{Pa s.}$$

(17) So:

$$0.1267 \text{ bar} \times \frac{10^5 \text{ Pa}}{1 \text{ bar}} \times \frac{7.5 \text{ torr}}{10^3 \text{ Pa}} \times \frac{1 \text{ mm Hg}}{1 \text{ torr}} \approx 95 \text{ mm Hg}$$

(18) From the ideal gas law:

$$PV = nRT \quad \text{so} \quad R = \frac{PV}{nT} = \frac{\text{atm.L}}{\text{mol.K}}$$

So the units of  $R$  are 'atm L K<sup>-1</sup> mol<sup>-1</sup>.'