SCIE1000, Solutions to Tutorial Week 5.

(1) 1,2. Here are some important points

- The approximate doubling time, T_2 can be calculated by $T_2 = \frac{70}{\%\text{ growth per unit time}}$. (Note that, for ease of calculation, the numbers 72 or 69 can be used instead of 72.)
- The **increase** over any doubling time is greater than the total of all preceding growth.
- In the time period when the amount of resource remaining is equal to the amount used in all history up to that point, only one doubling time remains before the resource is used up.
- Thus, if growth rates continue unchanged, resources can be exhausted before most people we even realise we are about to run out.
- Don't believe any prediction of the life expectancy of a non-renewable resource until you have confirmed the prediction by repeating the calculation.
- Once half all possible reserves of a certain resource have been consumed, the consumption rate can only decrease from that point.
- Stopping population growth is necessary for sustainability.
- It is our responsibility as members of a democracy to think.

3. In 2002, Professor Bartlett predicted that the price of a ticket would be \$80 in the year 2003, and \$160 in 2013, if the annual growth rate of 7% in prices were to be sustained. By 2016, if the predictions were correct, the ticket price would have been \$192. This is pretty similar to the actual price of \$179.

(2) This is a discussion question.

(3) Answers to this question will change from year to year. The broad structure of the answer will remain unchanged, but the specific values in the answer will change. The following values apply specifically to 2014.

1. If daily consumption remains unchanged (i.e. the growth rate is 0), then the proven oil reserves should last $\dfrac{1481526 \text{ Million Barrels}}{32480 \text{ Million Barrels/year}} = 45.6$ years.
2. These oil reserves may not last this long. As pointed out in the video, if the rate of consumption continues to increase, then these resources will be consumed much quicker. However, if this dwindling supply of oil causes humanity to shift to alternative forms of energy, the oil reserves will last longer. Similarly, as the oil supply becomes more scarce, it will become more valuable, which means fewer people will be able to afford it, and consumption should drop.

   Perhaps the most likely outcome is that bulk of this resource will be consumed in much less than 45.6 years, but because of other factors (such as pricing) and technologies, this resource probably won't be completely used up for a long time after that.
3. We are using an exponential growth model, so our model has the form $C(t) = Ae^{kt}$, where $C(t)$ is oil consumption as a function of time, and A and k are unknown constants.

   When $t = 0$ we have $C(0) = Ae^{k\times 0} = A$, and we know that $C(0)$ is the current consumption of oil rate, 32480 MBPY, so $A = 34280$ MBPY. In 2030, when $t = 18$ we have $C(18) = 32480e^{18k}$, and we know that $C(18) = 37960$ MBPY, so rearranging this equation gives

   $$\ln\left(\frac{37960}{32480}\right) = 18k$$

   so $k = 0.00866$ yr^{-1}. Thus, our model for the consumption of oil is $C(t) = 32480e^{0.00866t}$.
4. Rearranging this equation gives

   $$Q + \frac{C}{k} = \frac{C}{k}e^{kT}$$
   $$\frac{Qk}{C} + 1 = e^{kT}$$
   $$\ln\left(\frac{Qk}{C} + 1\right) = kT$$

   So, $T = \dfrac{1}{k}\ln\left(\dfrac{Qk}{C} + 1\right)$.

5. Substituting in values to the model we made in Part 4:

$$T = \frac{1}{k}\ln\left(\frac{Qk}{32480}+1\right) = \frac{1}{0.00866}\ln\left(\frac{1481526 \times 0.00866}{32480}+1\right) = 38.4 \text{ yr}.$$

6. The model that assumes no growth predicts that current oil reserves will be exhausted in 45.6 years, whereas the model that accounts for a growth in consumption predicts it will be exhausted in only 38.4 years. It is expected that the model that accounts for growth rate will predict a shorter time until the resource is exhausted. The difference is not that much though; both answers are the same order of magnitude. This is because the growth rate, 0.00866 yr^{-1} is quite small, and the amount of time the model is predicting over is not that long, only 18 years.

(4) Consider an exponential model, $N(t) = N_0 e^{kt}$, for some arbitrary quantity $N(t)$ which is growing exponentially, with initial amount N_0. Let T_2 be the doubling time. Then $N(T_2) = 2N_0$, since N_0 is the initial amount. Substituting this into our model gives

$$N(T_2) = 2N_0 = N_0 e^{kT_2}$$

cancelling out the N_0 gives

$$2 = e^{kT_2}$$

and rearranging for T_2 gives

$$T_2 = \frac{\ln(2)}{k}.$$

Now, the growth rate is $r = 100k$, so $k = \frac{r}{100}$. Substituting this into our expression for the doubling time gives

$$T_2 = \frac{100\ln(2)}{r}.$$

Finally, we note that $\ln(2) \approx 0.693$ so $100\ln(2) \approx 69.3$ and

$$T_2 \approx \frac{69.3}{r} \approx \frac{72}{r}$$

which is the formula for the Rule of 72.

(5) Answers to this question will change from year to year. The broad structure of the answer will remain unchanged, but the specific values in the answer will change. The following values apply specifically to 2014.

1. The current annual percentage growth rate of the Australian population (G_A) is:

$$\begin{aligned} G_A &= \text{Net migration rate} + \text{Birth rate} - \text{Death rate} \\ &= 1.127\% \end{aligned}$$

2. The equation for exponential growth is:

$$P(t) = A\,e^{kt}$$

In this case, t is the number of years from now, A is the current population and k is the previously calculated growth rate. Thus, an expression for Australia's population is:

$$P(t) = 22015576\,e^{0.01127t}$$

3. The Rule of 72 predicts the doubling time to be $T_2 = \frac{72}{1.127} = 63.9$ years. Let T_2 be the doubling time. Then $P(T_2) = 2 \times 22015576 = 44031152$. Using the exponential model $P(t) = 22015576\,e^{0.01127t}$, we have that $P(T_2) = 44031152 = 22015576\,e^{0.01127t}$ which implies

$$2 = e^{0.01127T_2}.$$

Rearranging for T_2 gives

$$T_2 = \frac{\ln(2)}{0.01127} = 61.5 \text{ years}.$$

These answers are very similar, which indicates that the Rule of 72 is a good approximation in this case.

4. One possible solution is

```
# A program to estimate the future population of Australia
from pylab import *

#Asks the user for the number of months for which
# the population should be calculated
years = eval(input("How many years to calculate Australia's population? "))
Pop0 = 22015576
Pop = Pop0
i=0

while i<=years:
    # Calculate and print the population for this year
    Pop = Pop0 * exp(0.01127*i)
    print("After", i, "years, pop'n is", Pop, "people.")
    i=i+1
```

5. Japan has a much higher proportion of older people in their population, partially because they have a longer life expectancy and partly because Japan has had a low birth rate for many years, so there is a lower proportion of younger people. Thus, the higher death rate comes about due to the higher proportion of older people in the population (as it is more likely for an older person to die than a younger one).

(6) Answers to this question will change from year to year. The broad structure of the answer and the program will remain unchanged, but the specific values in the answer will change. The following values apply specifically to 2014.

First we must calculate a function which models China's population and GDP any time t years from now. As we are assuming that the current growth rates remain constant, we may use an exponential equation to model these quantities. The equation for exponential growth is:

$$P(t) = A\,e^{kt}$$

In this case, t is the number of years from now, A is the current population and k is the growth rate. Thus, an expression for China's population, $P_C(t)$ is:

$$P_C(t) = 1343239923\,e^{0.00481t}$$

and an expression for China's GDP, $GDP_C(t)$ is:

$$GDP_C(t) = 11.3\,e^{0.092t}$$

with units in trillions of US dollars. We can now write a program. Here is one solution.

```
# A program to estimate the future population, GDP and GDP per
# capita of China
from pylab import *

# Helpful message for user
print("This program estimates the population, GDP and GDP per capita of China")
print("for the next 25 years.")

# Initial data
Pop0 = 1343239923
GDP0 = 11.3 # in trillions of US dollars
GDPcap0 = GDP0/Pop0*1000000000000 # is US dollars
i=0
```

```
years=25

print()
print("Year       Pop       GDP     GDP/cap     GDP x larger     GDP/cap x larger")
print("                   ($tn US)   $US pp")
print("-------------------------------------------------------------------------")
while i<=years:
# Calculates the population, GDP and GDP per capita for this year
    Pop = Pop0 * exp(0.00481*i)
    GDP = GDP0 * exp(0.092*i)
    GDPxLarger = GDP/GDP0
    GDPcap = GDP/Pop*1000000000000 # is US dollars
    GDPcapxLarger = GDPcap/GDPcap0
    # Prints the data for this year
    print("  ",i,round(Pop),"  ",round(GDP), " ",round(GDPcap),
     "     ",round(GDPxLarger,2), "              ",round(GDPcapxLarger,2))
    i=i+1
```

The output of this program is

```
This program estimates the population, GDP and GDP per capita of China
for the next 25 years.

Year       Pop       GDP     GDP/cap     GDP x larger     GDP/cap x larger
                   ($tn US)   $US pp
-------------------------------------------------------------------------
   0 1343239923.0    11.0   8412.0       1.0               1.0
   1 1349716471.0    12.0   9179.0       1.1               1.09
   2 1356224246.0    14.0   10015.0       1.2               1.19
   3 1362763398.0    15.0   10928.0       1.32               1.3
   4 1369334080.0    16.0   11923.0       1.44               1.42
   5 1375936443.0    18.0   13009.0       1.58               1.55
   6 1382570640.0    20.0   14195.0       1.74               1.69
   7 1389236824.0    22.0   15488.0       1.9               1.84
   8 1395935149.0    24.0   16899.0       2.09               2.01
   9 1402665772.0    26.0   18438.0       2.29               2.19
   10 1409428846.0    28.0   20118.0       2.51               2.39
   11 1416224529.0    31.0   21951.0       2.75               2.61
   12 1423052979.0    34.0   23951.0       3.02               2.85
   13 1429914352.0    37.0   26133.0       3.31               3.11
   14 1436808808.0    41.0   28514.0       3.63               3.39
   15 1443736506.0    45.0   31111.0       3.97               3.7
   16 1450697606.0    49.0   33946.0       4.36               4.04
   17 1457692271.0    54.0   37038.0       4.78               4.4
   18 1464720660.0    59.0   40412.0       5.24               4.8
   19 1471782938.0    65.0   44094.0       5.74               5.24
   20 1478879267.0    71.0   48111.0       6.3               5.72
   21 1486009811.0    78.0   52494.0       6.9               6.24
   22 1493174736.0    86.0   57277.0       7.57               6.81
   23 1500374207.0    94.0   62495.0       8.3               7.43
   24 1507608392.0    103.0   68189.0       9.1               8.11
   25 1514877456.0    113.0   74401.0       9.97               8.84
```

(7) Again, the numbers will vary from year to year. The following answer is for 2014, based on the answers to the previous question.

1. According to the output of our model, the Chinese GDP in 25 years time will be 112.7 trillion US dollars, which is $\frac{112.7}{11.3} = 9.97$ times the current Chinese GDP.
2. According to the output of our model, the Chinese GDP per capita in 25 years time will be 74400 US dollars, which is $\frac{74400}{8400} = 8.86$ times the current Chinese GDP per capita.
3. A. As the Chinese GDP per capita is predicted to increase to almost 9 times the current level and is increasing at an exponentially increasing rate, individuals in China would likely consume at an exponentially increasing rate, and consequently produce greenhouse gas at an exponentially increasing rate. This suggests that the Keeling curve over the next 25 years will look more like the prediction made by the exponential model in the lecture notes. Certainly, emissions are likely to rise very substantially.

   B. The higher population and wealth of Chinese individuals will place a large demand for resources on top of the current demand. As the supply of resources like food, coal and oil are limited, this will drive up the price of those resources. The high demand for these resources means that only a few wealthy countries and individuals will have access to them, severely restricting the global availability of these resources.
4. These predictions are not the most likely outcome. Our assumption that the current growth rates will remain constant is probably flawed. Cessation of China's one child policy may well increase growth rates, although increased levels of wealth and education may reduce growth rates. Also, if China's growing economy does have the effect of making resources more scarce and expensive, then it is unlikely that the current economic growth rate will be maintained. Furthermore, the Chinese economy is going through a period of very high growth, as China becomes a developed country. Once China becomes a developed country, the economy will likely settle down to levels of growth seen in other developed countries, at a rate closer to say 3% or less per annum. Thus, the actual growth trends probably won't be nearly as drastic as the model predicts.
5. Some points you may discuss
   - Many people consider having children as a fundamental human right.
   - Enforcing a limit on the number of children per family could have effects on the gender ratio of the next generation, as pregnancies with babies of unwanted gender may be terminated.
   - The previous point could be applied to any physical characteristic of a child.
   - There may be religious or cultural reasons to have a family of a certain size.
   - In poorer regions, a large family may be needed to ensure the family can earn enough money to survive.
   - A limit on the number of children a family has is a more humane way to curb population growth compared to other methods of population control, such as famine, disease and war.
   - Something needs to be done to curb population growth. As the video makes clear, population growth which consumes finite resources cannot be sustained.

(8)

1. Our model will be of exponential growth, so it will have the form $P(t) = P_0 e^{kt}$, where $P(t)$ is the population of Niger at any time t, t is the number of years since 2000 and P_0 is the population at time $t = 0$. From the table, we can see that P_0 is 10.9 million people. We can use the information from 2010 to calculate k.

$$\begin{aligned} 15.6 \times 10^6 &= 10.9 \times 10^6 e^{10k} \\ \log\left(\frac{15.6}{10.9}\right) &= 10k \\ 0.03585 &= k \end{aligned}$$

   So our model becomes $P(t) = 10.9 \times 10^6 e^{0.03585t}$.
2. The "Rule of 72" says that the doubling time, T_2, is given by

$$T_2 \approx \frac{72}{r}$$

   where r is the growth rate k as a percentage. So the approximate doubling time of the Nigerien population is $\frac{72}{3.585} \approx \frac{70}{3.5} = \frac{140}{7} = 20$ years.
3. The year 2090 is 80 years after the year 2010, which is 4 doubling times. If the population continues to grow at an exponential rate, the population at 2010 of 15.6 million will double 4 times:

$$\begin{aligned}2 \times 15.6 \times 10^6 &= 31.2 \times 10^6\\ 2 \times 31.2 \times 10^6 &= 62.4 \times 10^6\\ 2 \times 64.4 \times 10^6 &= 124.8 \times 10^6\\ 2 \times 124.8 \times 10^6 &= 249.6 \times 10^6\end{aligned}$$

So the approximate Nigerien population in the year 2090, assuming the same exponential growth rate is maintained is about 250 million people.

4. The UN prediction is less than the prediction from our model, so the UN prediction must not assume the growth rate stays constant. There are several factors which would cause the growth rate to decrease. There could be a carrying capacity for the Nigerien population; competition for resources mean that supporting large families in Niger could be come tougher as the population increases.

   Another reason the growth rate could decrease is that as the population of Niger ill probably become more highly educated, and the use of family planning services could become more popular. Similarly, as technological developments occur, Nigeriens may no longer need to support themselves by having large families.

(9) Part 2 involves calculations, but Part 1 may require a little bit of thought. The question says that "one has a carrying capacity and the other increases indefinitely". To identify which has the carrying capacity, it may be easier to identify which increases indefinitely. The function $5\sqrt{t}$ is a species area curve, which we know increases indefinitely, so the other must have the carrying capacity.

At the end of these solutions there is an example of a hand-written answer that would receive full marks on an exam.

(10) Suppose that after both populations grow for t hours, the population sizes are equal. Therefore:

$$\begin{aligned}1000 \times e^{0.03\times t} &= 3000 \times e^{0.01\times t}\\ \frac{e^{0.03\times t}}{e^{0.01\times t}} = e^{0.02\times t} &= \frac{3000}{1000}\\ \ln e^{0.02\times t} = 0.02t &= \ln(3)\\ t &= \frac{\ln(3)}{0.02} \approx 54.9 \text{ hrs}\end{aligned}$$

Therefore, in approximately 55 hours the two populations of bacteria will be equal.

(11) 1.

$$\frac{\Delta P}{\Delta t} = \frac{10^5 - 10^4}{5 - 0} = \frac{9 \times 10^4}{5} = 1.8 \times 10^4.$$

   Therefore, the average rate of change between $t = 0$ and $t = 5$ is 1.8×10^4 bacteria per hour.

2. To estimate the number of generations n between $t = 0$ and $t = 5$, we can solve the equation:

$$\begin{aligned}10^5 &= 10^4 \times 2^n\\ 10 &= 2^n\\ \ln(10) &= n \times \ln(2)\\ \therefore\ n &= \frac{\ln(10)}{\ln(2)} \approx 3.3\end{aligned}$$

   Therefore there have been approximately 3.3 generations between $t = 0$ and $t = 5$.

3. Since there were 3.3 generations between $t = 0$ and $t = 5$, the doubling time is $5/3.3 \approx 1.5$ hours.

4. There are two ways of doing this. If you are awake, you will notice that from $t = 0$ to $t = 5$ the population increased by a factor of 10, to 10^5. Hence from $t = 5$ to $t = 10$ it will increase by a factor of 10 again, giving a population of 10^6. Similarly, at $t = 15$ it will be 10^7, and at $t = 20$ it will be 10^8.

Alternately, using calculations, we proceed as follows. To calculate the time for the population to reach 10^8 individuals, we must first calculate the number of generations it takes to reach this number, and then multiply the result by the doubling time we just calculated. Therefore:

$$\begin{aligned} 10^8 &= 10^4 \times 2^n \\ 10^4 &= 2^n \\ \ln 10^4 &= n \ln 2 \\ \therefore n &= \frac{\ln 10^4}{\ln 2} = 13.3 \end{aligned}$$

Therefore, 13.3×1.5 hours $= 19.95 \approx 20$ hours. So the population will reach 100 million individuals in approximately 20 hours.

(12)
1. If we take $\log_{10}$ of each side, we have $\log_{10} S = \log_{10} C + p \log_{10} a$. On a log/log graph (with x values being $\log_{10} a$ and y values being $\log_{10} S$), this is a straight line, with y-intercept $\log_{10} C$, and slope equal to p.
2. Working from the $\log$ version of the equation in Part 1 and the coordinates of the two points on the line, we have:
   $1 = \log_{10} C + p \times 0$ and $2 = \log_{10} C + p \times 2$.
   Solving these equations gives: $\log_{10} C = 1$ so $C = 10$; and $p = 1/2$.
   Hence the equation is: $S(a) = 10a^{0.5}$.

(13)
1. At time 0, the concentration of H ions is 10^{-8} mol L^{-1}.
   At time 2, the concentration of H ions is 10^{-7} mol L^{-1}.
   Hence the average rate of change in concentration of H ions is:
   $$\frac{10^{-7} - 10^{-8}}{2 - 0} = 4.5 \times 10^{-8} \text{ mol L}^{-} \text{ min}^{-1}.$$
2. The concentration of H ions must increase by $10^{-5} - 10^{-7} = 99 \times 10^{-7}$ mol L^{-1}, at a rate of 4.5×10^{-8} mol L^{-1} min^{-1}, which takes 220 more minutes. Hence this will have occurred at time $t = 222$ min.

(14)
1. The average rate of change is:
   $$\Delta = \frac{10^4 - 10^5 \text{ mg}}{8 - 2 \text{ hrs}} = \frac{-9 \times 10^4 \text{ mg}}{6 \text{ hrs}} = 1.5 \times 10^4 \text{ mg/hr}.$$
   Therefore, the average rate of change is 1.5×10^4 mg per hour.
2. Suppose that we call $t = 2$ our 'initial' time, we therefore can say that initially we have 10^5 mg of the material. Using this, and the fact that 6 hours later (i.e. $t = 8$) we have 10^4 mg of the material, we can find the decay constant:
   $$\begin{aligned} A(t) &= A_0 e^{-kt} \\ 10^4 &= 10^5 e^{-6k} \\ \frac{1}{10} &= e^{-6k} \\ \ln(\frac{1}{10}) &= -6k \\ \therefore k &= \frac{\ln(10)}{6} \\ &\approx 0.384 \end{aligned}$$
   Therefore, the decay constant of the material is $k \approx 0.38$ per hr. Using this, we can obtain the half life of the substance:
   $$\begin{aligned} \frac{1}{2} &= e^{-kt_{\text{half life}}} \\ \ln(\frac{1}{2}) &= -0.38 t_{\text{half life}} \\ \therefore t_{\text{half life}} &= \frac{\ln(2)}{0.38} \approx 1.8 \text{ hrs} \end{aligned}$$

The half life of the material is approximately 1.8 hours.

3. We can use a trick here based on what we know about radioactive (and all exponential) decay. We know that the time for a substance to decay a certain by fraction is always fixed. The information at the start of the question states that the substance decayed from 10^5 mg to 10^4 mg in 6 hours. Using this fact, if we undergo another two more order of magnitude decays (i.e. a 90% decay), which we know will take 12 hours (two, six hour decays) the time when the material reaches 10^2 mg is $t = 20$.

(15) 1. Let t_e be the time when both objects have the same temperature, and let that temperature be T. Also let $T_{0,1}$ be the initial temperature of the first object, $T_{0,2}$ be the initial temperature of the second object, C_1 be the temperature of the first oven, and C_2 be the temperature of the second oven. Then we have that

$$C_1 - (C_1 - T_{0,1})e^{-kt_e} = T = C_2 - (C_2 - T_{0,2})e^{-kt_e}.$$

Subbing in values gives

$$70 - (70 - 60)e^{-0.1t_e} = 60 - (60 - 80)e^{-0.1t_e}$$

and rearranging gives

$$10 = 30e^{-0.1t_e}$$

and

$$\frac{\ln \frac{1}{3}}{-0.1} = t_e = 10.986min$$

2. The initial difference in temperature of the object and environment is 40^oC. The object cooled by half the difference in temperature between the object and the oven in 10 minutes. Since the object cools exponentially, the decay in temperature is occurring at a constant rate. Thus, it will take another 10 minutes for the difference in temperature to halve again. This time the temperature difference is only 20^oC, so after 20 minutes, the temperature will be 70^oC. Another 10 minutes later for the difference in temperature to halve again, and the current difference in the temperature of the object and oven is 10^oC, so after 30 minutes, the temperature of the object will be 65^oC.

(16) Don't just wait to receive the answer to this question. Think about it. It essentially boils down to: we assume that the *proportion* of tagged fish will remain constant, on the reef and in the fish that are captured. Does that help? We know three of the values used to calculate the proportions...

1. Let: S_1 be the number of tagged fish in the pond, S_2 be the number of fish caught during the second catch, and S_3 be the number of tagged fish in the second catch of fish. If we assume the proportion of tagged fish in the pond does not change over the time between catches then we can assume that the proportion of tagged fish in the second sample is an accurate representation (and equal to) the proportion of tagged fish in the entire pond. Therefore, using this equality:

$$\begin{aligned} \frac{S_1}{N} &= \frac{S_3}{S_2} \\ \therefore\ N &= \frac{S_1 \times S_2}{S_3} \end{aligned}$$

2. The patent measures plasma volume (and hence blood volume), by:
   - introducing a known volume of detectable, harmless, biodegradable over time, large-molecule substance into the individual's bloodstream;
   - waiting for sufficient time for the substance to diffuse throughout the blood stream;
   - taking a blood sample of known volume from the patient;
   - measuring the concentration of the substance in the blood sample; and
   - using the two known volumes and the measured concentration to estimate the total volume.

3. Both Parts 1 and 2 are closely related to the notion of concentrations. The approach in Part 1 is effectively assuming that the 'concentration' c_1 of tagged fish in the total population equals the 'concentration' c_2 of tagged fish in the second catch, so $c_1 = c_2$. The value of c_2 can be calculated immediately, and calculating c_1 requires only the known number of tagged fish, and the unknown total population. The equation can be solved for the single unknown value (the total population).

In Part 2, the approach is effectively the same. Let c_2 be the concentration of the detectable substance in the blood sample, which can be measured, and let c_1 be the concentration of the substance in the blood stream. Again, we have $c_1 = c_2$, and calculating c_1 requires only the known volume of substance introduced into the bloodstream and the unknown total blood volume. Once again, the equation can be solved for the single unknown value (the blood volume).

The answers to Question (9).

3. (a) $M_2 = 5\sqrt{t}$ → Species area curve

⇒ grows indefinitely.

Hence M_1 must have carr. capac.

(b) $M_1 = 10 \Rightarrow 20(1 - e^{-0.2t}) = 10$

$\Rightarrow 1 - e^{-0.2t} = \frac{1}{2}$

$\Rightarrow e^{-0.2t} = \frac{1}{2}$

$\Rightarrow -0.2t = \ln(0.5) \Rightarrow t = \frac{\ln(0.5)}{-0.2}$

$= 3.47$ months

$M_2 = 10 \Rightarrow 5\sqrt{t} = 10$

$\Rightarrow \sqrt{t} = 2$

$\Rightarrow t = 4$ months.